10. Model Discretization and Calibration

Lesson Objectives:

1) Define various model discretization types and select the appropriate discretization for the analysis objective, model accuracy, and computational efficiency.
2) Describe and select models for various materials within structural analysis (metals and confined versus unconfined concrete).
3) Outline the procedure and various assessments to calibrate a numerical model using experimental results.
4) Explain how an eigenvalue analysis can be utilized to verify a numerical model including such quantifiable parameters as mass participation factor and modal assurance criteria.
5) Outline the concept of a nonlinear static pushover curve and describe how it can be used to assess a numerical model.

Background Reading:

1) Read ______Elnashai and Sarno Chapter 4 (recommended, but not required)______________

Model Calibration Introduction:

1) Construction of any analytical model requires validation.
    a. Verify the structural system idealization is reasonable.
    b. Level of desired accuracy is obtainable.
    c. The salient structural response mechanisms are captured.
        i. Cracking in concrete
        ii. Crack moment of inertia for concrete members
        iii. Post-yield strain hardening
        iv. Force-displacement hysteretic response
2) Models can be calibrated and validated in numerous ways.
3) Examples include (list is not inclusive):
    a. Experimental response calibration
    b. Eigenvalue analysis

   c. Nonlinear static pushover curves

4) Before discussion on model calibration, let's quickly outline the types of model, materials, and elements that may be encountered in analysis practices.

Model Classifications:

1) Structural models are numerical idealizations to simulate the response characteristics of systems.
2) Under dynamic loads, generally three levels of models exist. In order of increasing complexity, accuracy, and computational demand (Figures 1 and 2):
   a. Substitute or equivalent single degree-of-freedom systems (SDOFs)
      i. Also known as macroscopic models
   b. Stick models representing multiple degree-of-freedom systems (MDOFs)
   c. Detailed models representing multiple degree-of-freedom systems (MDOFs)
      i. Also known as microscopic models
3) Note stick models are sometimes referred to as hybrid or intermediate models.
4) Substitute or equivalent SDOF models: structure is idealized as an equivalent SDOF system.
   a. Four parameters are needed: effective mass (m_{eff}), effective height (h_{eff}), effective stiffness (k_{eff}), and effective damping (ζ_{eff}).
   b. The equivalence used to estimate stiffness and damping assumes that the displacement of the original structure and substitute model is the same.
   c. For inelastic systems, oftentimes the secant stiffness may be assumed to a target displacement as well as the damping set to the equivalent viscous value.
   d. Examples include elastic-perfectly plastic, hardening, and strain-softening behaviors.
   e. Often used for spectral and response time history analyses.
   f. Inadequate to capture or represent the local response of a structure.
   g. Effective for global analysis with low computational demand.
5) Stick models of MDOF systems: elements idealize a number of members of the prototype structure. In a building structure, each story is represented by a single line of finite element

representing the deformational characteristics of the columns and beams.

   a. The lateral stiffness of each "stick element" is the stiffness of the frame comprising the columns and beams.
   b. For dynamic analysis, the masses are also lumped. However on occasion, distributed mass can be used to simulate the response of a structural shear wall.
   c. Suitable for sensitivity studies of design and analysis, such as the beam-to-column strength ratio, irregularities, etc.
   d. Inadequate to capture the distribution of ductility demands and damage among individual structural members.

6) Detailed models of MDOF systems: structural systems are discretized into a large number of elements with section analysis or spatial elements in either 2D or 3D.
   a. Modeling approach permits the representation of member geometry and history of stresses and strains at fibers along the member length or across the section dimensions.
   b. Adequate to provide global response quantities and the relationship between local and global response.
   c. Beams and columns are generally modeled by flexural elements.
   d. Braces are generally modeled by truss elements.
   e. Shear and core walls by 2D elements such as plates and shells.
      i. However sometimes simplifications are made for a grillage model.
   f. For some structures, 3D modeling may be required for sufficient accuracy.
   g. Essential modeling to represent stress concentrations, local damage patterns, and/or the interface between different materials.
   h. Can be computationally demanding, particularly for large structures when inelastic time history response is required.
7) The selection of the type of modeling will permit the appropriate level of discretization while considering the target of the analysis (what level of details are desired) as well as the accuracy and computational demand.

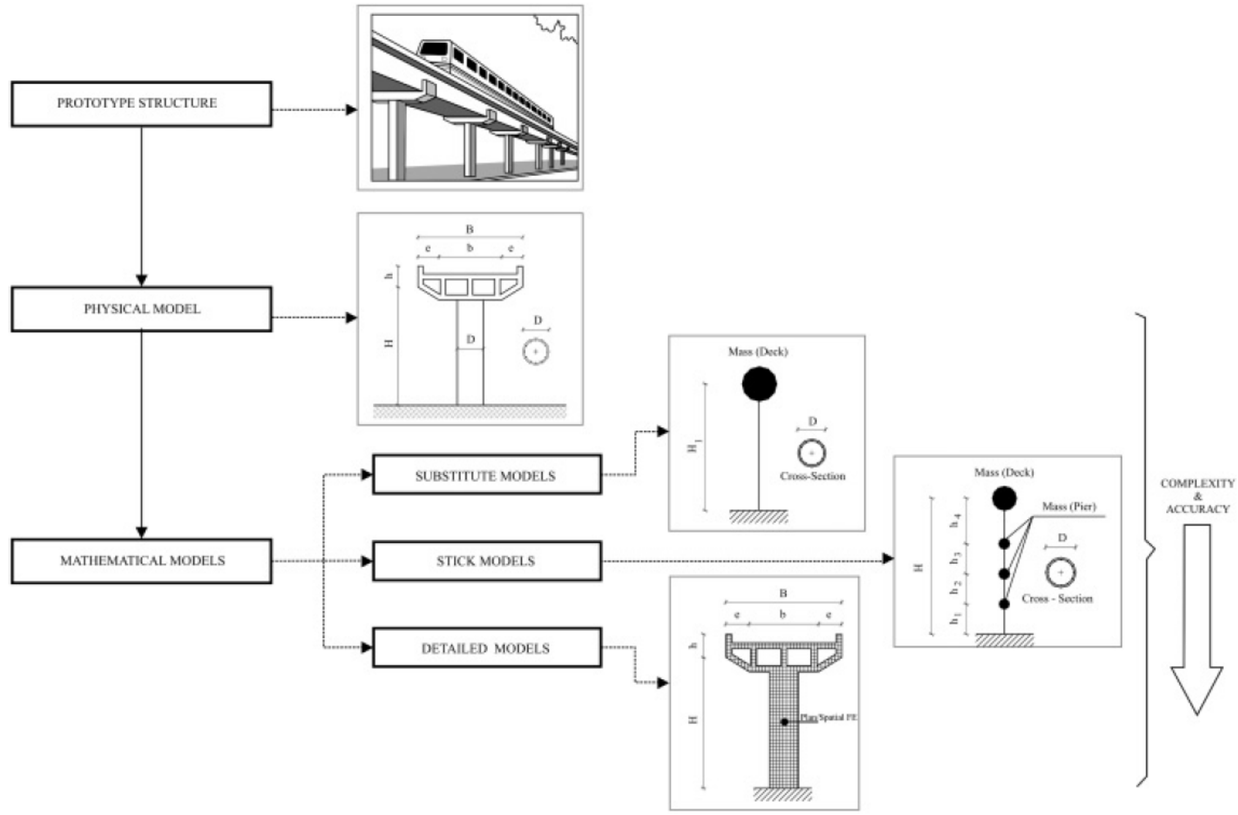

Figure 1.Various levels of structural models for dynamic loads[1].

Model type	Discretization type	Tridimensional effects	Structure prototype	Analysis target	Complexity/ Accuracy	Computational demand
Substitute	SDOF	Usually not accommodated	Primarily regular structures	Global response	Low	Low
Stick	MDOF	Accommodated	All types of structure	Global response	Medium	Medium
Detailed	MDOF	Accommodated	All types of structure	Local and Global response	High	High

Figure 2. Qualitative summary of various model types.

[1] Figures obtained from: Elnashi A.S. and Di Sarno, L. (2008) *Fundamentals of Earthquake Engineering: From Source to Fragility*. 2nd Edition. Wiley, West Sussex, UK. 469 pp. (unless otherwise noted).

Materials within Structural Models:

1) Two basic materials using in civil structures are metals and concrete.
2) Let's examine the possible material models by material type.

Metals and steel:

1) For metallic materials, common uniaxial elastic plastic materials are selected to perform elastic and inelastic analysis of structures (Figure 3).
    a. Linear elastic perfectly-plastic (LEPP)
    b. Linear elastic perfectly-plastic with strain hardening (LESH)
    c. Linear elastic with non-linear hardening (LENLH)
    d. Power laws: Menegotto-Pinto (MP) and Rambery-Osgood (RO)
        i. Able to capture significant non-linear strain hardening for aluminum alloys and stainless steels
2) For these uniaxial elastic plastic models, three identified shortcomings are:
    a. Presence of a horizontal yield plateau followed by a strain-hardening zone
    b. Reduction in strain-hardening slope with increase in the strain amplitude
    c. Experimentally observed cyclic hardening and softening
3) To account for these aforementioned shortcomings, more complex models are most appropriate.
4) However these elastic-plastic models are adequate for moderate ductility demands of approximately 2-4.

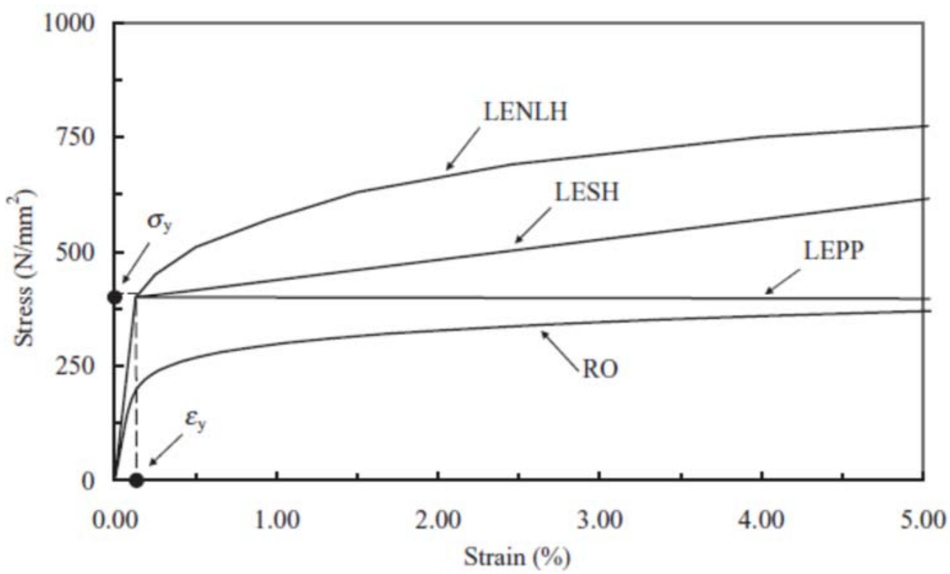

Figure 3. Uniaxial models for various metallic stress-strain relationships.

Properties	Model types			
	Linear elastic-perfectly plastic	Linear elastic-plastic strain hardening	Linear elastic-plastic non-linear hardening	Power Law (Ramberg–Osgood model)
Constitutive equations*	$\sigma = E\varepsilon$ $\sigma = E(\varepsilon - \lambda)$	$\sigma = E\varepsilon$ $\sigma = \sigma_y + E_t\left(\varepsilon - \frac{\sigma_y}{E}\right)$	$\sigma = E\varepsilon$ $\sigma = k\varepsilon^n$	$\varepsilon = \frac{\sigma}{E} + a\left(\frac{\sigma}{b}\right)^n$
Yield criterion	$\sigma = \sigma_y$	$\sigma = \sigma_y$	$\sigma = \sigma_y$	$\sigma = \sigma_{y,0.2}$
Hardening type	Null	Linear	Non-linear	Non-linear
Pros and Cons	Easy to implement No spreading of plasticity Not suitable for stress-controlled inelastic analysis Suitable for mild steel	Easy to implement Spreading of plasticity Suitable for stress-controlled inelastic analysis Suitable for mild steel	Easy to implement Gradual spreading of plasticity Experimental data for calibration Suitable for mild steel	Implementation more time-consuming Suitable for high inelasticity (alloy metals) Experimental data for calibration

Key: * = the first equation holds for $\sigma \leq \sigma_y$ and the second for $\sigma > \sigma_y$; λ = non-zero scalar; a, b and n are material constants that can be computed through laboratory tests.

Figure 4. Elastic plastic model types for metals.

Reinforced Concrete:

1) Reinforced concrete is a complex heterogeneous material.
2) To describe the material behavior up to and beyond the ultimate capacity may require the following the following (refer to Figure 5):
    a. Nonlinear stress-strain relationship
        i. Macroscopic model examples include:
            1. Linear elastic models with zero-valued tension
            2. Nonlinear elastic models
            3. Elastic plastic models
    b. Fracture and failure surface
    c. Post-fracture response and failure criteria
        i. Different assumptions here will yield very different results
    d. Model for reinforcement steel
        i. See previous section
    e. Bond-slip and interface characterization
        i. Often ignored due to lack of reliable data

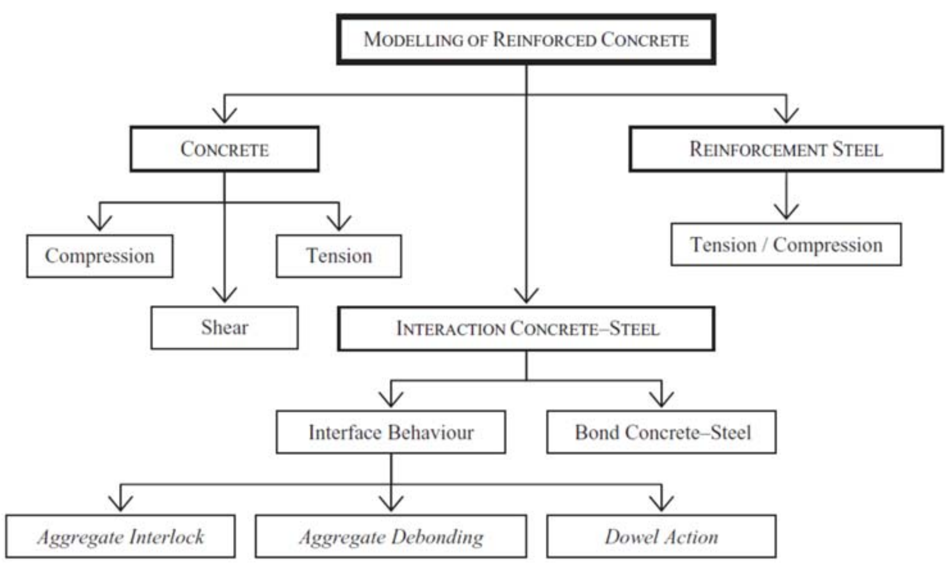

Figure 5. Modeling hierarchy for reinforced concrete members without torsion.

3) Stress strain curves with a section of a reinforced concrete element are a function of the

amount of transverse reinforcement.

4) With increased transverse reinforcement, increased ductility capacity is created as a result of lateral confinement. The common model to account for the confined concrete is Mander's (1988) as illustrated in Figure 6.
    a. Often times at within the plastic hinge of a member, the core will be modelled as "confined" and the shell or cover as "unconfined" or "plain concrete".

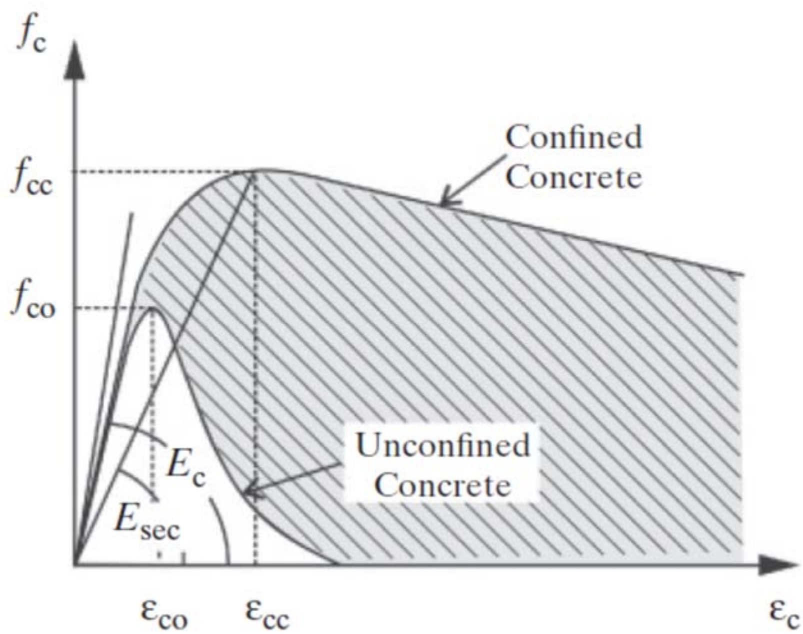

Figure 6. Mander's model for confined concrete.

Sectional Analysis:

1) Geometric and material properties of components are modeled using cross-sections.
2) Dependent on the type of section being utilized, composite or different material properties are often required.
    a. One example is a reinforced concrete section.
    b. For an RC and composite section, it is necessary to define the area and location of the reinforcement steel.
3) Three formulations for sectional analysis within finite element include:

a. Fiber (or filament) models:
   i. Most reliable formulation and form the basis of distributed plasticity models.
   ii. An example RC section is shown Figure 7.
b. Phenomenological (or mathematical) models:
   i. Requires calibration of numerous parameters to simulate the monotonic and hysteretic response of cross-sections under different loadings.
   ii. An example moment curvature is shown in Figure 8.
c. Mechanical (or sectional spring) models:
   i. Generally used for lumped plasticity and discretized as a number of springs.
   ii. Can also combine some features of the fiber models for lumped plasticity.
      1. Example plastic hinges at member ends (Figures 9 and 10).

4) If considering lumped plasticity, the length of the plastic hinge is required for analysis.
5) One common method is based on Pauley and Priestley's (1992) model:

$$l_p = 0.08L + 0.15 f_y d_b \quad [in, ksi]$$

a. L = length of the beam in inches
b. f_y = yield strength of the longitudinal reinforcing steel in ksi
c. d_b = bar diameter in inches

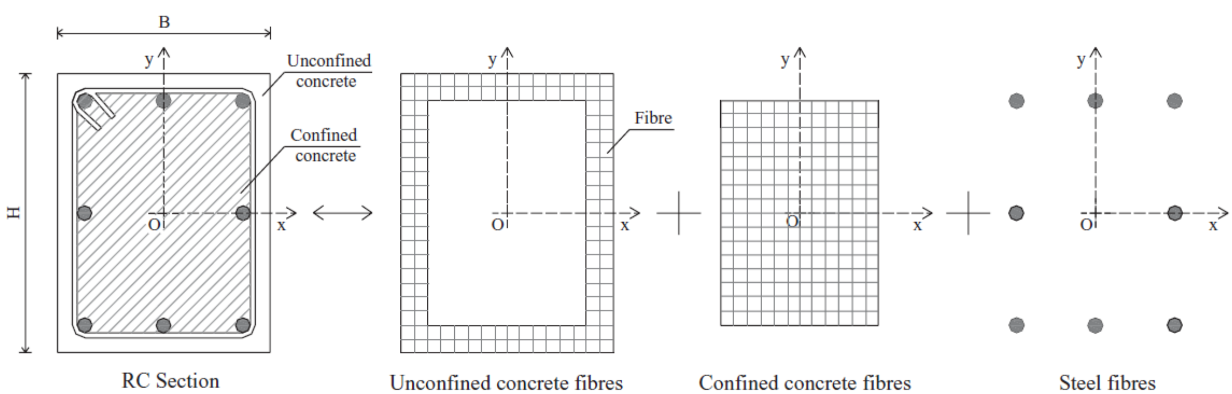

Figure 7. Reinforced concrete section discretized as a fiber section.

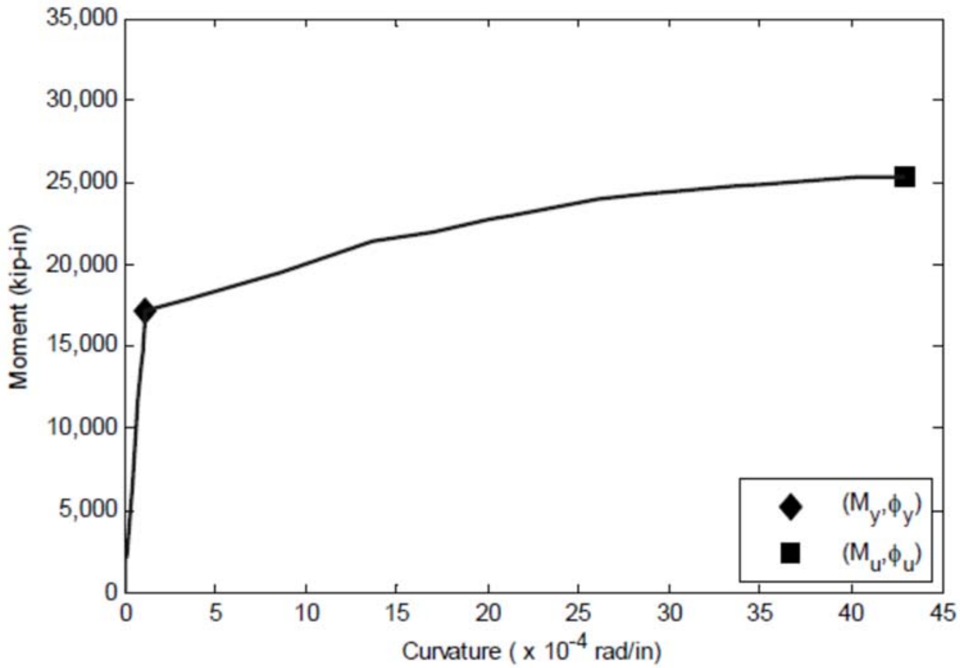

Figure 8. Example sectional analysis: moment curvature relationship for a reinforced concrete member[2].

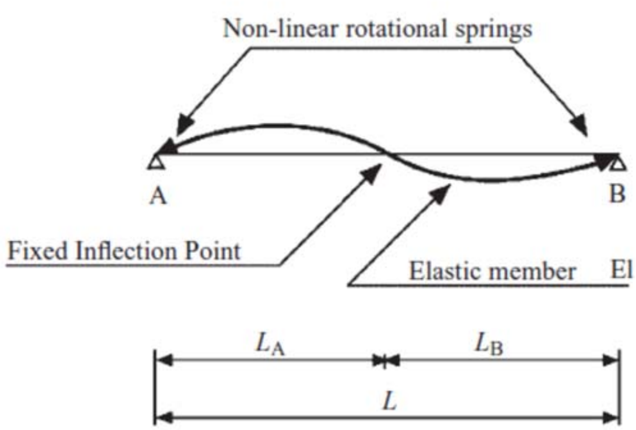

Figure 9. Schematic of a lumped plasticity element. Note commonly used for beams and columns.

[2] Figure obtained from: Wood, R.L., Hutchinson, T.C., and Hoehler, M.S. (2009). Cyclic Load and Crack Protocols for Anchored Nonstructural Components and Systems. Structural Systems Research Project Report Series. SSRP 09/12. UC San Diego.

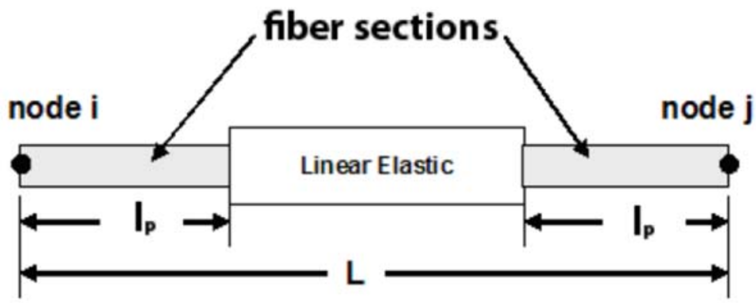

Figure 10. Alterative schematic of a lumped plasticity element with fiber section ends[3].

Model Calibration - Experimental Results:

1) When available, calibration of a numerical model can be performed against experimental data.
2) Often this is performed at the element level – specific beam, column, or beam-column joint.
    a. However this is not an explicit limitation.
3) In the assessment of the representation of a numerical model, various comparisons can be made:
    a. Initial tangent stiffness
    b. Secant stiffness to a point of interest
    c. Yield point
    d. Strain hardening
    e. Maximum capacity and displacement
    f. Post-peak strain hardening and stiffness
    g. Unloading stiffness
    h. Residual drift
    i. Hysteretic energy
4) An example of a numerical model calibration of a reinforced concrete shear wall is shown in Figure 11.

[3] Figure obtained from: McKenna, F., Fenves, G., and Scott, M. (2000). Open system for earthquake engineering simulation (OpenSEES), Pacific Earthquake Engineering Ressearch (PEER) Center, University of California, Berkeley, CA.

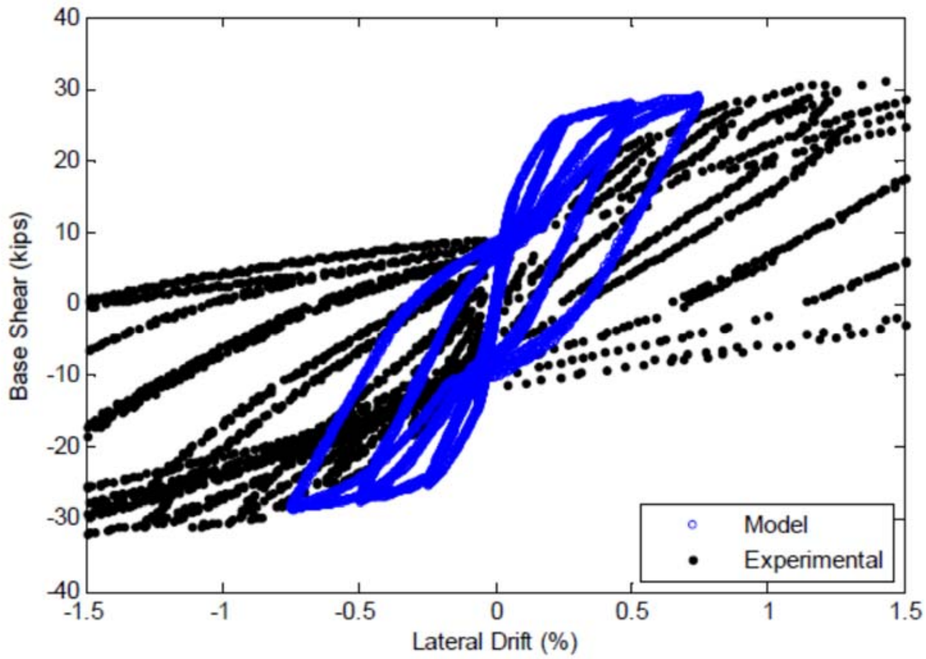

Figure 11. Example trial model calibration of a reinforced concrete shear wall[4].

Model Calibration - Eigenvalue Analysis:

1) An eigenvalue analysis can be carried out for a numerical model to determine its dynamic characteristics.
2) In this analysis, the fundamental periods and mode shapes can be obtained.
3) Note this indicates the distribution of the mass and stiffness of a structural system at its initial elastic state.

Mass Participation Factor:

4) In addition, the mass participation factor can be determined to quantify the mass participating in a specific n^{th} mode.

[4] Figure obtained from: Wood, R.L., Hutchinson, T.C., and Hoehler, M.S. (2009). Cyclic Load and Crack Protocols for Anchored Nonstructural Components and Systems. Structural Systems Research Project Report Series. SSRP 09/12. UC San Diego.

$$\frac{M_n}{M_T} = \frac{\{\phi_n\}^T [M] \{\phi\}}{M_T}$$

a. M_n = modal mass in the nth mode
b. M_T = total mass of the structure
c. [M] = mass matrix
d. $\{\phi_n\}$ = nth mode shape

5) The mass participation is typically highest in the fundamental mode, with a rough approximate range of 70-90% for buildings. Note significant values in modes other than the first indicate the potential for the influence of higher modes on the structural response.
6) Note this is not indicative of its potential inelastic behavior, however a post-inelastic behavior eigenvalue analysis can be conducted in selective software packages.
7) Post-inelastic behavior eigenvalue analyses are indicative of the accumulated damage in terms of stiffness reductions (assuming that the mass is constant – which is normally the case).
    a. Quantify the period elongation of a structural system.

Modal Assurance Criteria:

8) To compare various mode shapes as obtained from different sources, the Modal Assurance Criteria (MAC) can be calculated.
9) The MAC provides a quantitative value to compare two mode shapes via the following expression:

$$MAC(\phi_1, \phi_2) = \frac{|\phi_1^H \phi_2|^2}{|\phi_1^H \phi_1||\phi_2^H \phi_2|}$$

    a. $\{\phi_n\}^H$ = complex conjugate and transpose of the nth mode shape
10) The higher the MAC value, the more alike the two mode shapes are.

11) MAC values can be used to evaluate the damage in a structural system by comparing the initial and the final mode shapes or to compare the modes as obtained by different structural identification techniques.

Model Calibration - Nonlinear Static Pushover Curves:

1) A common method to assess the structural performance in terms of strength and deformation capacity is to use a static nonlinear pushover analysis.
2) This assessment permits the estimation of the numerous structural parameters, including building drift, interstory drift, element deformation (plastic mechanisms), and internal forces.
3) This analysis accounts for the geometric and material nonlinearities as well as the redistribution of internal forces under a developed plastic mechanism.
4) To perform this analysis a lateral load pattern is defined and then incrementally increased until a target displacement or other criteria is achieved.
5) Various patterns can be defined, but the most common fall into:
    a. Triangular - best for elastic force range distribution
    b. Uniform – not common, but noted to represent the force distribution for a structure with significant damage
    c. Modal – combination of the first mode and selected other modes to account for the higher mode effects when larger period structures are considered.
        i. An example modal pattern is illustrated in Figure 12.

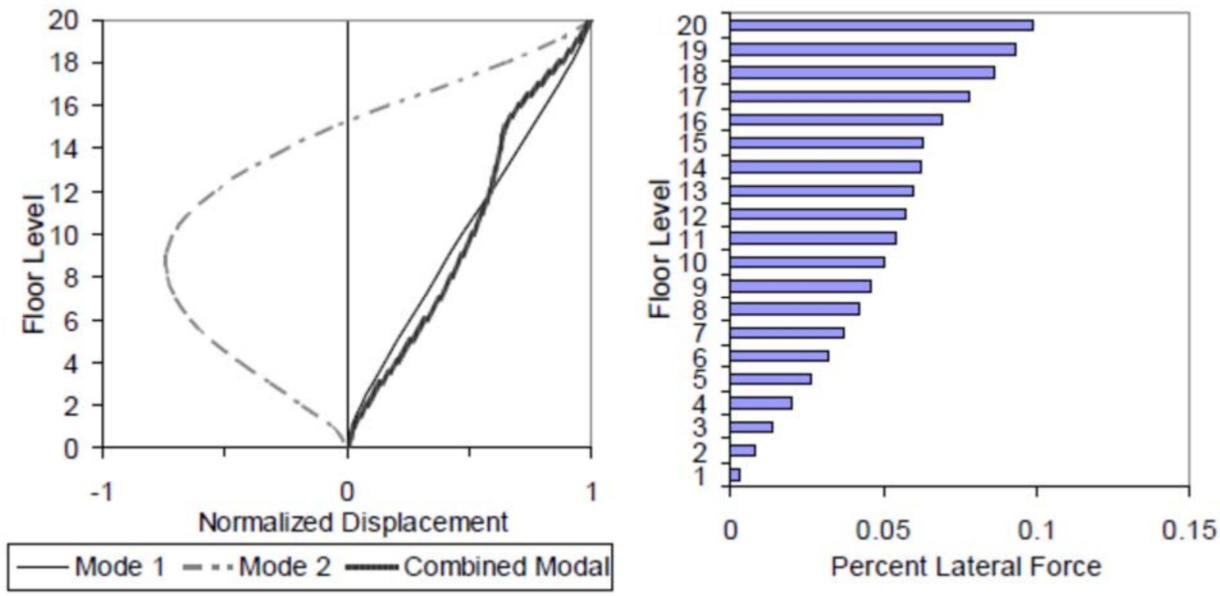

Figure 12. Example twenty story reinforced concrete SMRF: (a) first two mode shapes with a combined mode representation ($1.0\{\phi_1\}+0.20\{\phi_2\}$) and (b) normalized lateral force pattern for the nonlinear static pushover[5].

6) Nonlinear pushover curve illustrates the anticipated inelastic behavior of the structure.
    a. Often the base shear is compared to the total drift of the building.
    b. A normalized base shear quantity is illustrated in Figure 13, where it is divided by the weight of the structure. Note this corresponds to the seismic coefficient as previously outlined in the seismic design notes.
7) Nonlinear pushover curves can also be converted to capacity curves for equivalent inelastic SDOF systems.
    a. Refer to Chopra, A. K., & Goel, R. K. (2002). A modal pushover analysis procedure for estimating seismic demands for buildings. *Earthquake Engineering & Structural Dynamics*, 31(3), 561-582.

[5] Figure obtained from: Wood, R.L., Hutchinson, T.C., and Hoehler, M.S. (2009). Cyclic Load and Crack Protocols for Anchored Nonstructural Components and Systems. Structural Systems Research Project Report Series. SSRP 09/12. UC San Diego.

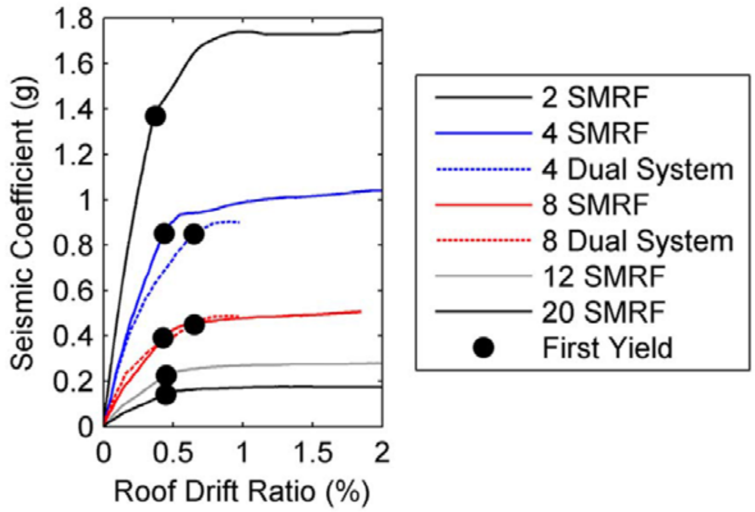

Figure 13. Example nonlinear static pushover for a seven building suite. Note these are reinforced concrete SMRF and dual systems (OMRF and structural shear wall) buildings[6].

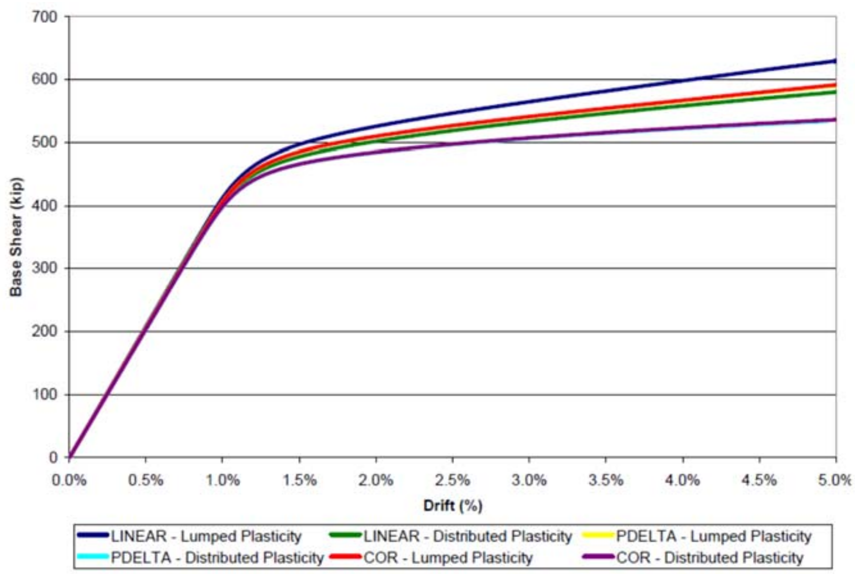

Figure 14. Example pushover curves for example numerical model assumptions of spread/lumped plasticities and geometric transformations: (a) Linear, (b) P-Delta, and (c) Corotational - large displacements and small strains[7].

[6] Figure obtained from: Wood, R.L. and Hutchinson, T.C. (2013) Crack Protocols for Anchored Components and Systems. ACI Structural Journal, 110(3): 391-402.
[7] Figure is self-made, not previously published.

8) However nonlinear static pushovers are far from perfect and has a few noted limitations:
    a. The response of a MDOF structure is directly related to an equivalent SDOF system. In dynamic time history response, the shape of the fundamental mode (and other mode shapes) will vary significantly on the level of inelastic and its distribution of plasticity.
    b. Caution should be used for deformation estimates for structures which higher mode effects are significant. The prescribed methods in codes (FEMA 356) often explicitly ignore the contribution of higher modes.
    c. Pushover are well-established for 2D models. Any three-dimensional and torsional effects are difficult to model and little work has been conducted on asymmetric 3D systems with irregularities in mass and/or stiffness.
    d. Any progressive stiffness degradation as a result of cyclic loading is not accounted for in the pushover analysis. In addition, it neglects duration and cumulative energy dissipation.
    e. This is a static method that concentrates on the strain energy of the structure. Neglected are other sources of energy dissipation associated with the dynamic processes (kinetic and viscous damping energies).
    f. Only horizontal spatial load distributed loads are explicitly account for in a pushover. The vertical component of an earthquake is neglected, which can be of great importance for certain structural systems.
9) Pushovers are attractive for design practice in lieu of nonlinear time history analysis, but caution should be used in its assessment.