Plane Frames

Lesson Objectives:

1) Derive the member local stiffness values for two-dimensional plane framed members.
2) Assemble the local member stiffness matrix into global member stiffness matrix.
3) Assemble the structural stiffness matrix using direct stiffness and code numbering techniques.
4) Outline procedure and compute the response of plane frames using the stiffness method.

Background Reading:

1) Read Read Kassimali – Chapter 6

Introduction:

1) What is a plane frame?
   a. Two-dimensional assemblage of ________________________________.
   b. Connected together by either ____________________________ connections.
   c. Subjected to external loads where the forces and couples lie in the plane of the structure.
   d. The members are subjected to ________________, ________________, and ______________.
      i. Neglect any __________________________ effects (____________).
2) Due to the developed member actions:
   a. ______________________________ invokes the stiffness relationships from ______________________________.
   b. ____ invoke the stiffness relationships from ______________________________.
3) In this section of notes, the analysis of rigidly connected plane frames is the focus. Released connections will be visited in the next section of notes.
4) Before a discussion of the member stiffness matrices, let's first detail an idealized model.

Analytical Model:

1) A plane frame can be modeled as a series of ______________________________ connected via ______________________.
    a. Subdivide a member if necessary such that the members are ____________ ________________________________.
2) Members are connected at each end to ________________________________.
3) The unknown external reactions only act at the ____________________________.
4) The joints are modeled as ________________________.
    a. That is, the corresponding ends of the adjacent members are rigidly connected to the joints.
    b. Satisfy the ______________ and ______________ conditions for the actual structure.
    c. The size of the joints are considered to be __________________________.
        i. Is this reasonable? ______________________________________
    d. If a certain degree of freedom needs to be eliminated, example for a hinge, then __________________________ are used as needed.
        i. This topic will be discussed in the next set of notes.
5) See the example illustrated within Figures 1-3.

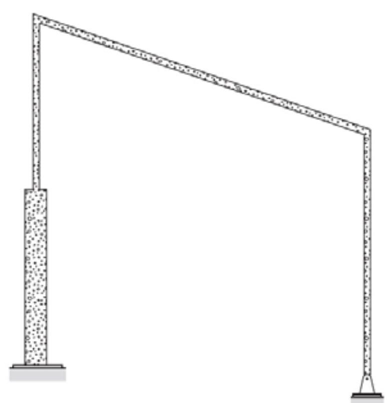


Figure 1. Physical diagram of plane framed structure[1].

[1] All figures in Plane Frames were modified from: Kassimali, Aslam. (2012). *Matrix Analysis of Structures*. 2nd edition. Cengage Learning.

Coordinate Systems:

1) The Global Coordinate System can be described as:
    a. Global coordinate system is a right-handed XYZ coordinate system.
    b. The X axis is oriented in the horizontal direction, where the positive direction is typically defined to the ___________________.
    c. The Y axis is oriented in the vertical direction, where the positive direction is typically _______________________.
    d. All external loads lie in the XY plane.
2) Origin is typically placed at the bottom leftmost joint of the structure (see example).
    a. Therefore all joints have ____.
3) The Local Coordinate System is oriented as:
    a. Local coordinate system is a right-handed XYZ coordinate system.
    b. The X axis is typically oriented along the member, where the positive direction is defined into the member and coincides with the centroidal axis of the member in the undeformed state.
    c. The Y axis is typically oriented in the vertical direction, where the positive direction is upwards.
4) Refer to Figure 2, for an example frame structure of ________________________.

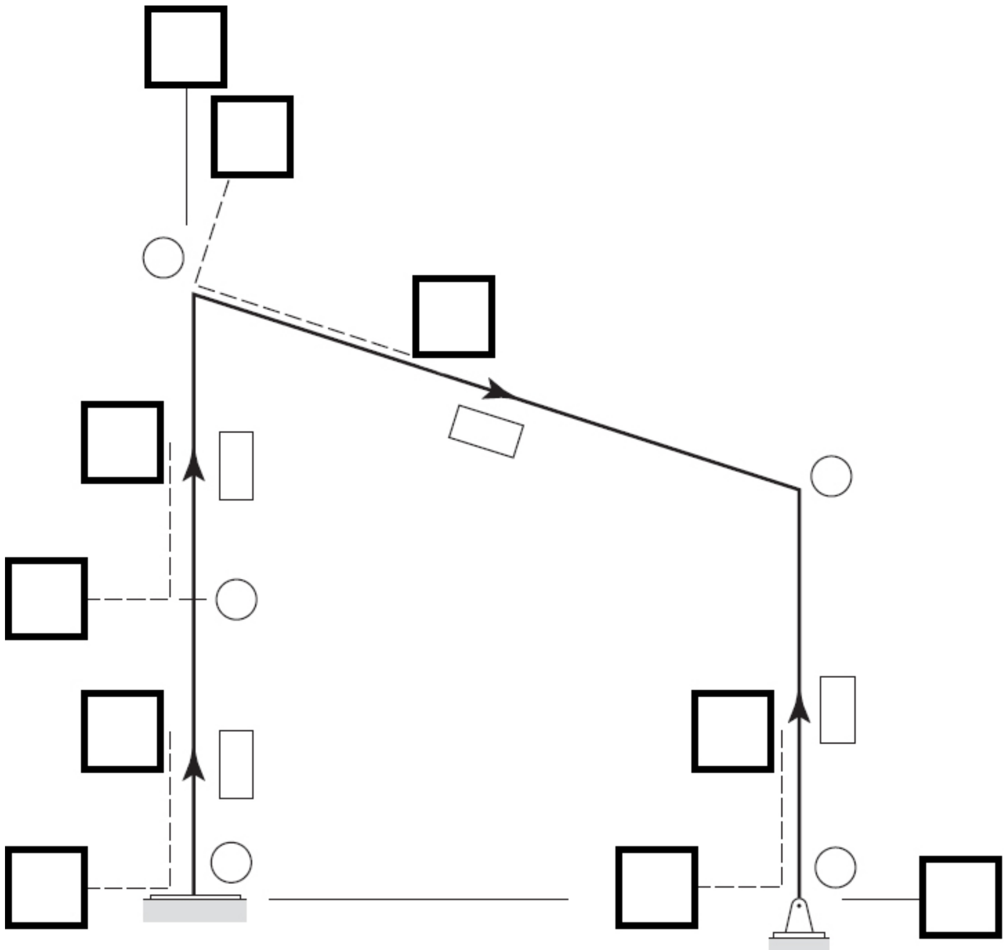


Figure 2. Discretized frame model identifying the two coordinate systems.

Degrees of Freedom (DOFs):

1) Identification of DOFs are **essential** for accurate structural analysis.
2) What are degrees of freedom:
    a. Independent joint deformation (translation, rotation) that are required to characterize the deformed shape of the structe under arbitrary loading.
    b. Also known as the degree of kinematic indeterminacy of a structure.
3) Frame structures: a joint can have up to three DOFs.
    a. ____
    b. ____
    c. ____

4) General formula appears as:

$$\#DOF = \#\,Joint\ Displacements\ of\ an\ Unsupported\ Structure \\ - \#\,Joint\ Displacmeents\ Restrained\ by\ Supports$$

$$NDOF = NCJT(NJ) - NR$$

5) For a plane frame structure, the formula simplifies to:

$$NCJT_{frame} =$$

6) Refer to Figures 3 and 4.

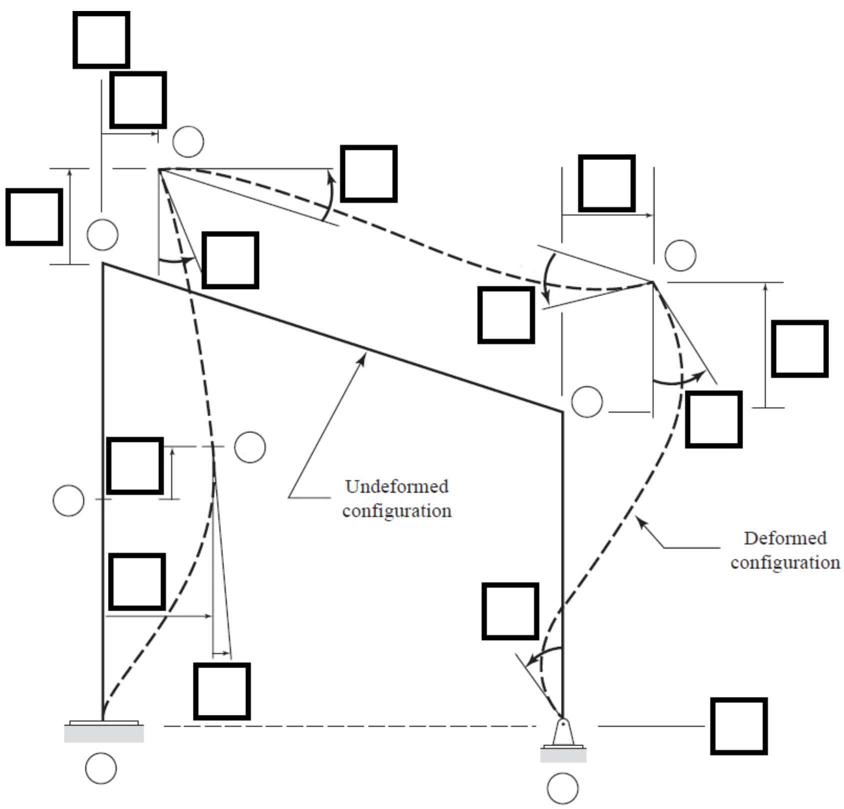


Figure 3. Discretized frame model identifying the degrees of freedom.

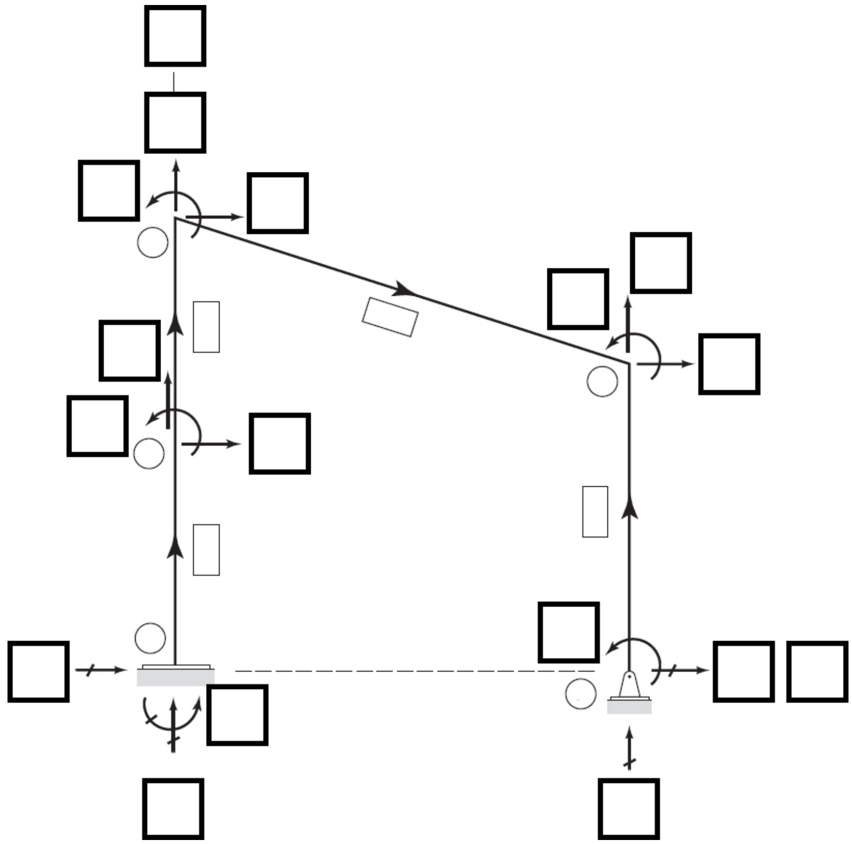


Figure 4. Discretized frame model identifying the structure coordinate numbers.

Degrees of Freedom (cont'd):

1) The text (chapters 3, 5, and 6) specificies an approach for identifying and numbering DOFs for a considered structure.
   a. Consistency leads to organization.
   b. Identity directly on numerical model using number arrows, where:
      i. Restraint are with an arrow and a slash.
      ii. This same approach is used for applied forces and reactions.
   c. Number starts at the lowest numbered joint and proceeds to the highest numbers, where:
      i. Free DOFS are identified first.
      ii. Identify DOFs in the following order:
         1. ____.
         2. ____.

3. ____.

iii. Upon completion of free DOFs labeling, continue routine for numbering restrained DOFs.

2) Resultant of DOF numbering is a joint displacement vector, ______.

Member Level Stiffness Relationship:

1) To develop the member stiffness relationship, let's examine an arbitrary member *m*.
2) When member *m* is subjected to external loads:
   a. The member will _____________________.
   b. ____ are induced at the member ends.
3) Refer below to Figure 5.

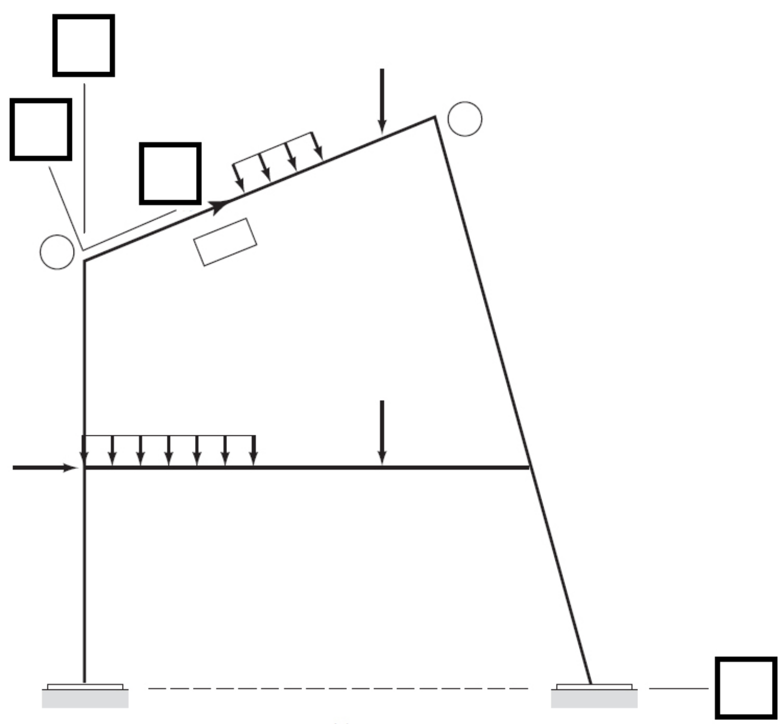


Figure 5. Example frame shown with external loading, note the location of member *m*.

4) The member has ___________ DOFs.

   a. Each _________________ is needed to specify the displacement of *m* member ends.

   b. Member forces and displacements are defined in the _______________________ ________________________.

   c. The DOFs are numbered in the same manner as done previously (see previous notes and reference Figure 6 below).

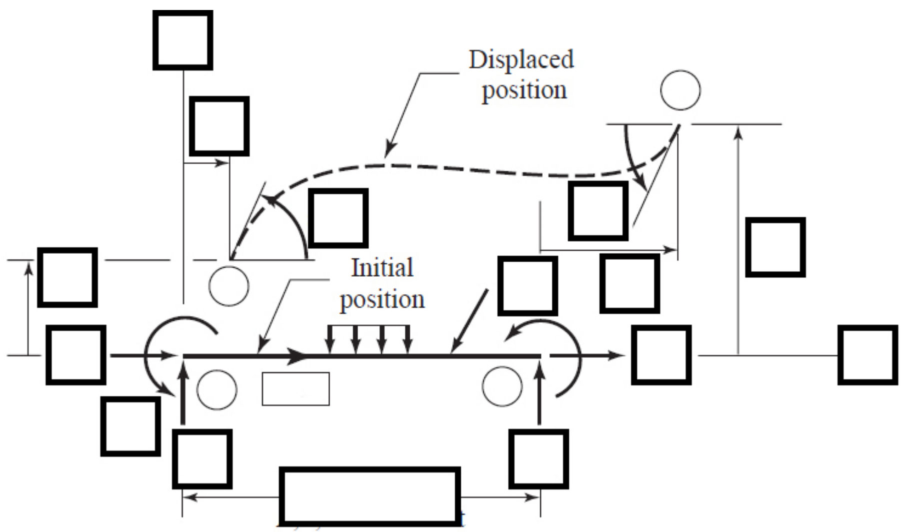


Figure 6. Member *m* shown with the member end forces and displacements in local coordinates.

5) ________________ are specified for each member to characterize its deformation.

6) The relationship between the __________________________________ ({Q}) and the __________________________________ ({u}) can be determined by subjecting the member to each DOF separately (keeping the other DOFS restrained) and fixed end forces due to external loading.

   a. This was done previously for both truss and beam structures.

   b. _________________, _________________, and _________________ were utilized to derive the stiffness coefficients.

7) The stiffness coefficients and the member fixed-end forces for a frame member are illustrated in Figures 7-13.

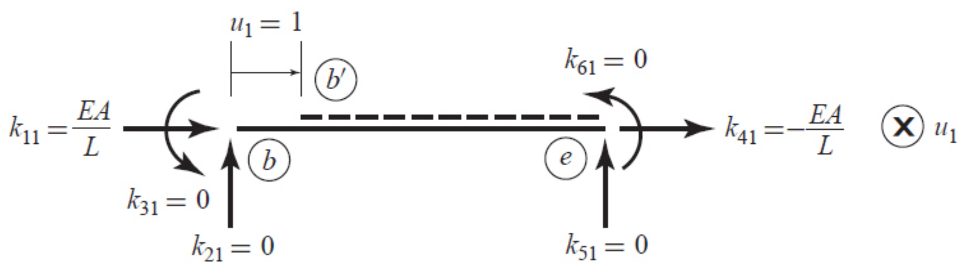


Figure 7. Stiffness coefficients for a frame member by imposed unit displacements: ________.

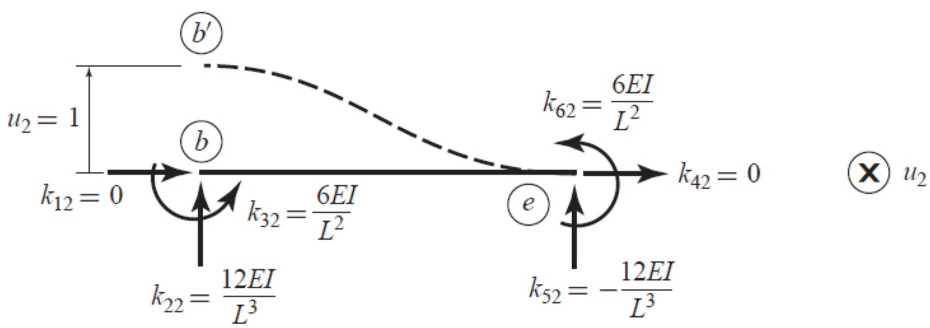


Figure 8. Stiffness coefficients for a frame member by imposed unit displacements: ________.

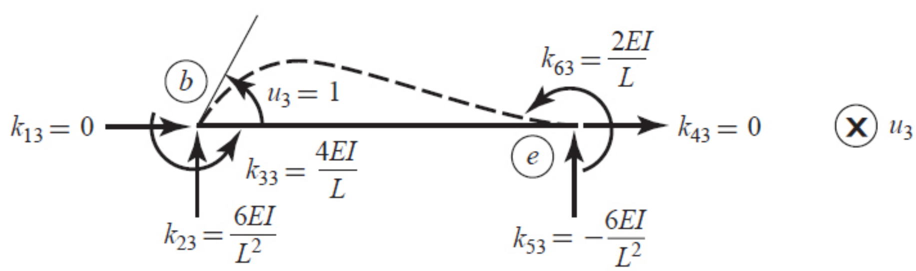


Figure 9. Stiffness coefficients for a frame member by imposed unit displacements: ________.

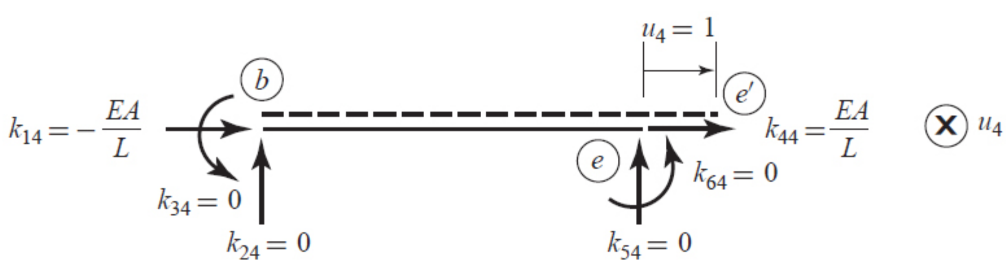


Figure 10. Stiffness coefficients for a frame member by imposed unit displacements: ________.

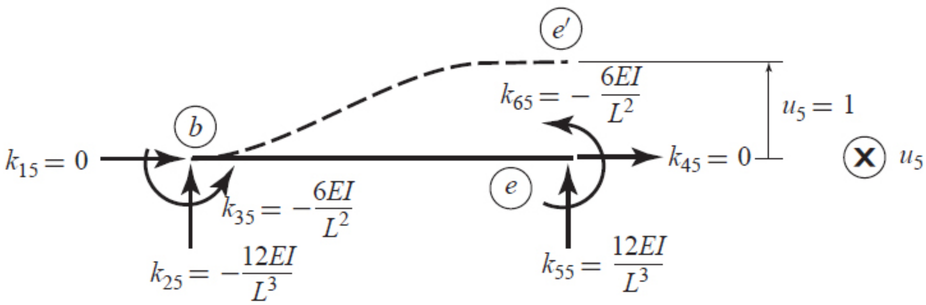


Figure 11. Stiffness coefficients for a frame member by imposed unit displacements: ________.

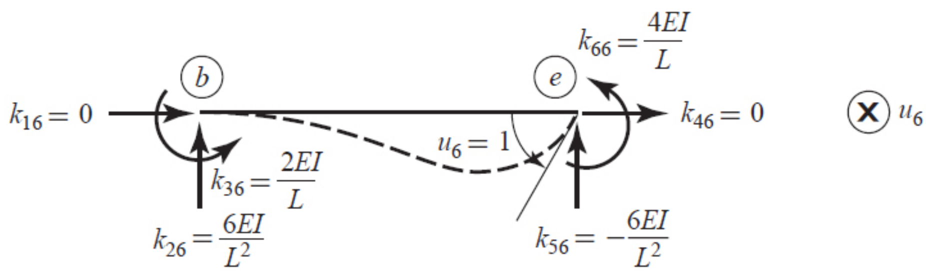


Figure 12. Stiffness coefficients for a frame member by imposed unit displacements: ________.

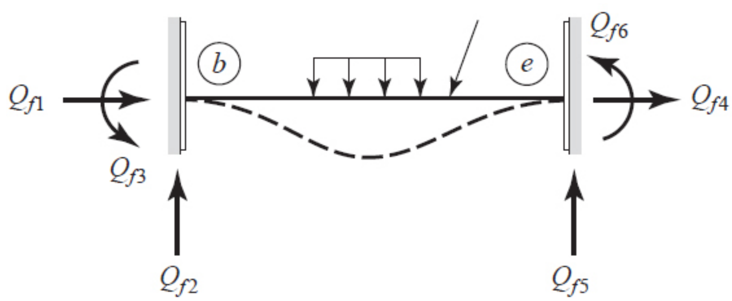


Figure 13. Relationship of ______________________________________ due to external loads applied along the member.

Member Stiffness Coefficients:

1) As outlined in previous notes, the stiffness coefficients can be derived.
2) Using these stiffness coefficients, the relationship between the member end forces and the end displacements can be written in matrix form as:

3) For a frame member, the local stiffness matrix is:

$$\boldsymbol{k} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

4) What is known about this local stiffness matrix?

   a. ______________________________

   b. ______________________________

   c. ______________________________

   ____

   ____

   ____

Member Fixed-End Vector in Local Coordinates:

1) Unlike beams, the loading for plane frame members can be ________________________ applied in ____________________________ in the plane of the structure.
2) Before calculation of the fixed-end forces, any loads acting on the member must be ____ _____________ into directional components aligned in the _______________ coordinate system.
3) Examine the example plane in Figure 14-15.
4) The vertical load _______ is acting downward. To resolve this into the local x and y directions:

5) The perpendicular loads and any applied couples can be calculated using fixed-end force equations as previously done for beams.
   a. __________________ and __________________ are tabulated.

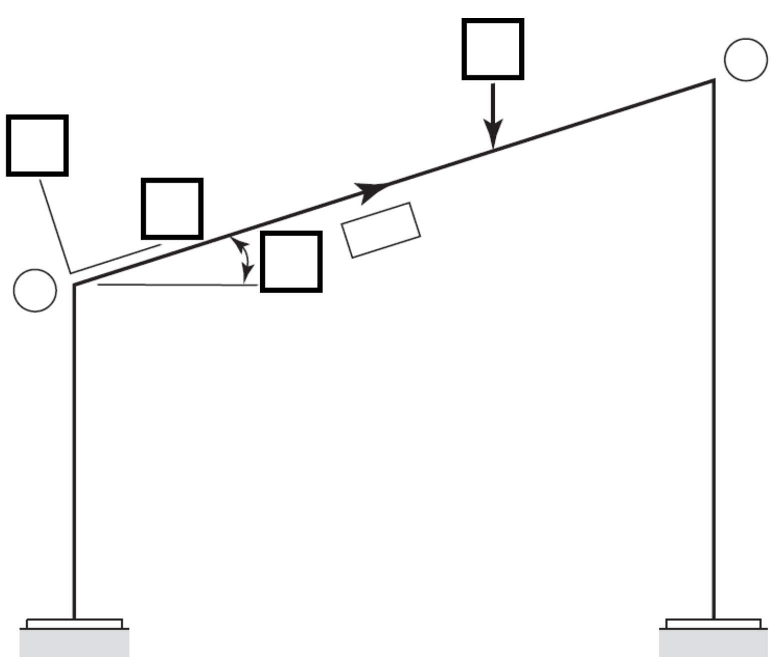


Figure 14. Identification of member ____ for the example inclined frame.

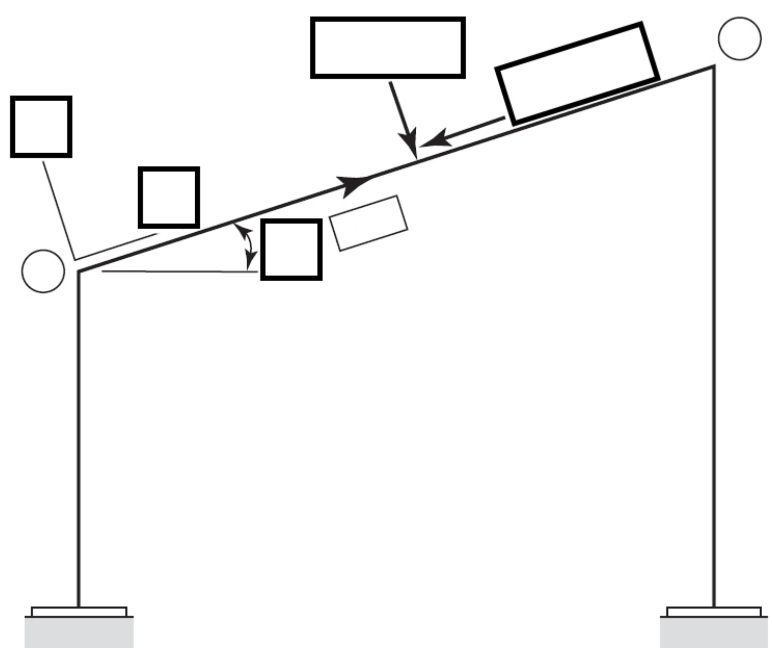
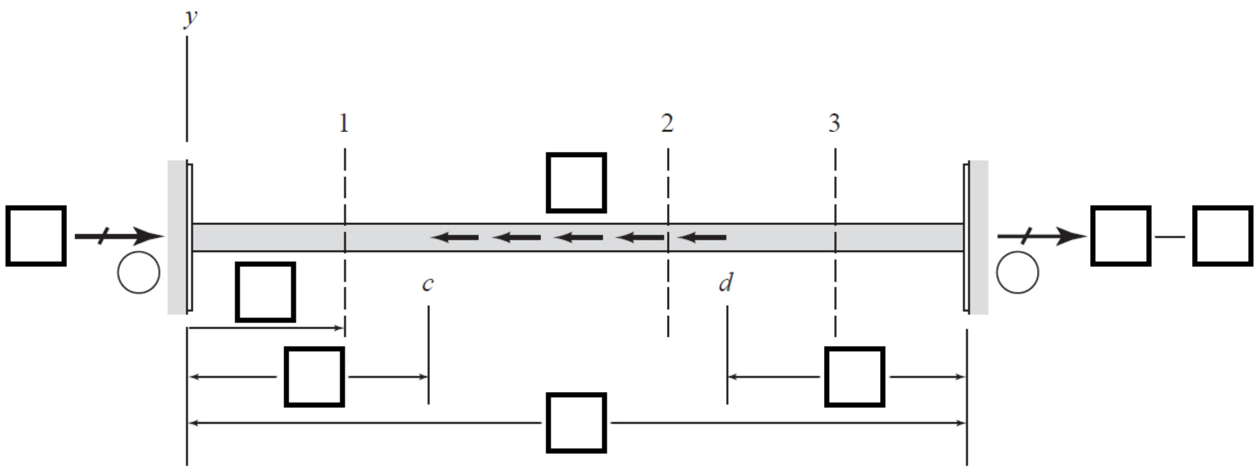


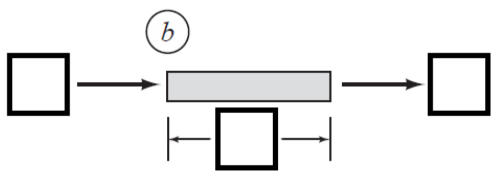
Figure 15. Force resolution of external loads acting on member m.

6) Expression for the parallel loading (___________________________) can be also calculated using the fixed-end force equations as tabulated.
7) The fixed-end equations for axially loads can be derived by _____________________ of the differential equations for member ____.
8) Consider a fixed-fixed member of a plane frame (Figure 16).
   a. Subjected to ____.
   b. Fixed-end axial forces develop at the member ends as shown in Figure 16.
      i. Note the sign convention.
9) Recall from mechanics that the axial strain and axial displacement (of the centroidal axis) of a member is:

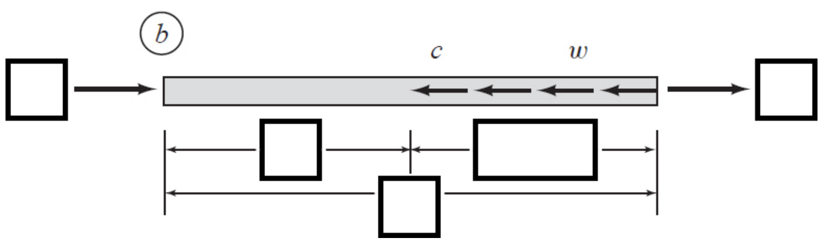
10) Through substitution into Hooke's law:



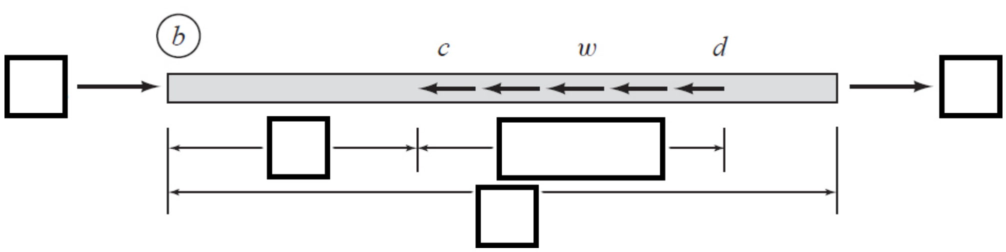
(a) Overview.



(b) Section cut at plane 1.



(c) Section cut at plane 2.



(d) Section cut at plane 3.

Figure 16. Example fixed-fixed member of a plane frame subjected to a uniformly distributed axial load w over a part of its length.

11) By multiplying by the cross-sectional area, an expression for the axial force can be written as:

12) This equation above represents the differential equation for a ________________ _____________________________________.

13) Sign convention: ________________ is considered positive.

14) The total axial deformation for a member can be determined by multiplying both sides of the equation by ____ and integrating the equation over the ____________ of the member.

15) Acknowledging that EA is constant for _____________________, the axial deformations can be expressed as:

Example Fixed-End Axial Forces:

1) Now, let's obtain the example fixed-end axial forces for the member shown in Figure 16.
2) The uniformly distributed axial load is applied over ______________________________.
3) The axial force ______ can be expressed as a piecewise function (______________________________).
    a. Therefore one can divide the member into three segments that correspond to the sections 1, 2, and 3.
    b. Namely: ________, _________, and _________.

4) By examination of the member from the left to section 1, one can express Q_a as:

5) By examination of the member from the left to section 2, one can express Q_a as:

6) By examination of the member from the left to section 3, one can express Q_a as:

7) Through substitution into the equation for the axial deformation:

8) Evaluating the boundary conditions, one can solve for FA_b:

9) By a summation of forces in the x-direction, one can find an expression for FA_e (for the example case in Figure 16):

Summary of Member Fixed-End Vector in Local Coordinates:

1) With knowing the fixed-end axial forces, the total fixed-end force vector (local coordinates) can be constructed.

2) Typical values for the fixed-end axial forces are also tabulated.
3) *Sign-convention*: for local fixed-end forces in local coordinates:
    a. Axial forces are positive when ______________________________________.
    b. Shear forces are positive when ______________________________________.
    c. Moments are positive when ____.
    d. Be mindful of the signs!

Coordinate Transforms:

1) Similar to trusses, the orientation of frame members vary.
2) Therefore it is critical to _______________ the member stiffness relations to the global coordinate system.
3) Note this section of notes is abridged, more details can be found in the text book and in previous truss notes.
4) Examine the example plane frame illustrated in Figure 17.
    a. The orientation of member m is defined by ________.
        i. This is measured ____ ____.
    b. When the frame is subjected to external loads, the member deforms where ____________________ and ____________________ develop at its ends.
    c. Figures 18 and 19 illustrate an arbitrarily displaced member m in the local and global coordinates, respectively.

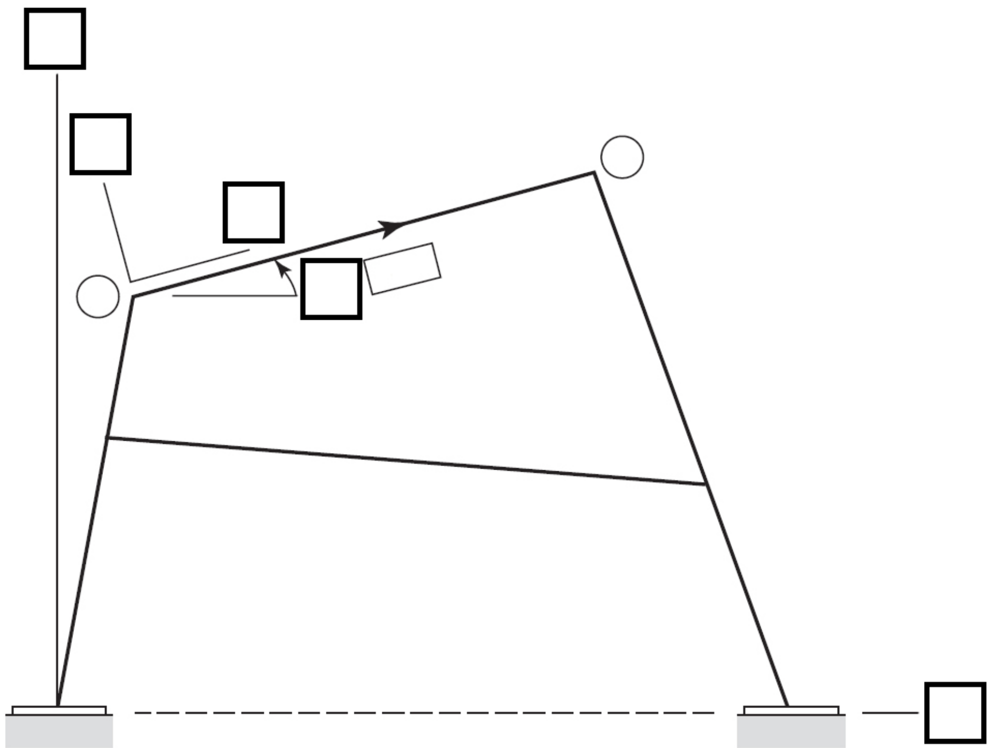


Figure 17. Location of member ____ for the example inclined frame.

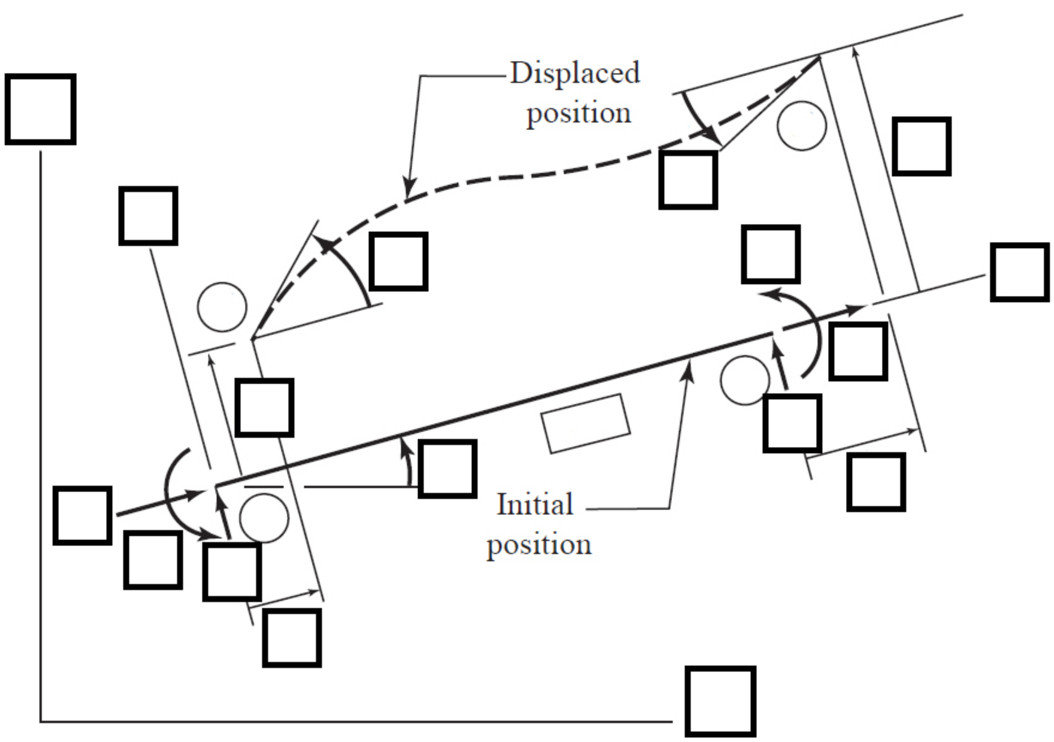


Figure 18. Member ____ in ____.

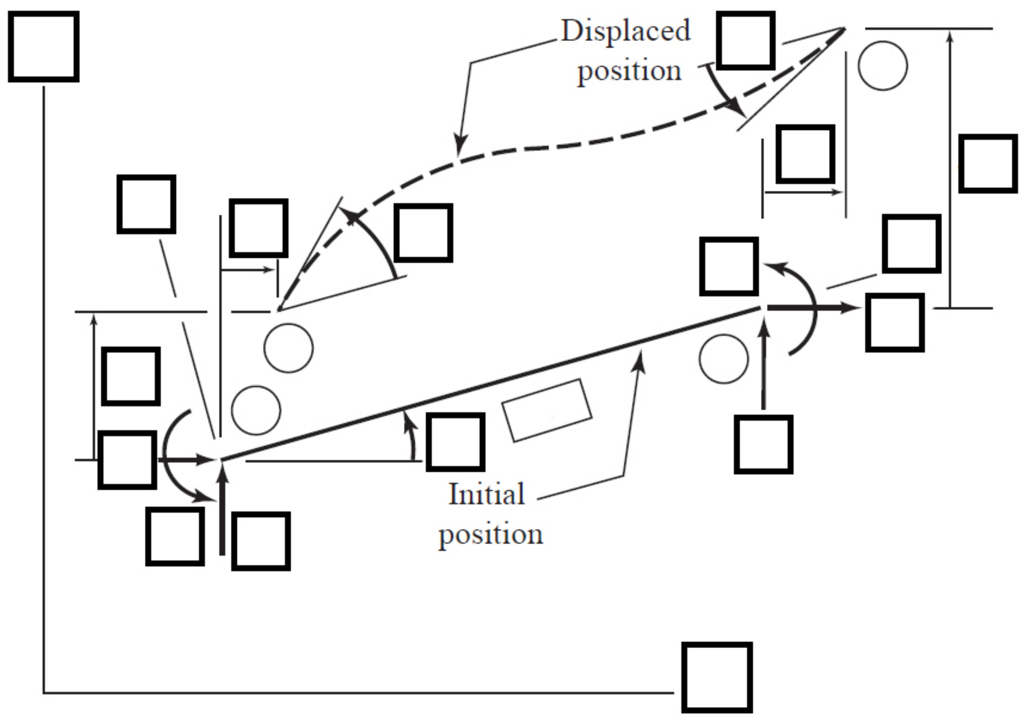


Figure 19. Member ____ in ________________________________.

5) By comparing Figures 18 and 19, the member end forces for each coordinate system can be related.
    a. The components of each coordinate system are related through ____________.
    b. Acknowledging that this relationship was previously defined, one can write an expression for the member end forces in local coordinates as:

$$\{Q\} = [T]\,\{F\}$$

$$[T] = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 & 0 & 0 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\theta) & \sin(\theta) & 0 \\ 0 & 0 & 0 & -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

c. Since the member end displacements are defined in the same manner as the member end forces, the relationship for the displacements in local coordinates is:

6) To express the reverse relationship, one can write:

$$\{F\} = [T]^T \{Q\}$$

$$[T]^T = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & 0 & 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

a. Since the member end displacements are defined in the same manner as the member end forces, the relationship for the displacements in global coordinates is:

7) This transformation matrix is similar to that derived for plane trusses.

Member Stiffness Relations in Global Coordinates:

1) In establishing the stiffness relationships in global coordinates for plane frames, the similarities between plane trusses and beams are noted.
2) Recall the member stiffness relation and substitute into the force transformation relations:

3) Recall the member end displacement transformation relation and substitute into the equation to write:

4) The member global stiffness matrix is now-coupled, that is a global stiffness coefficient can be a function of _____, _____, _____, _____, and _____.
    a. Typically it is easier to find the local member stiffness and transform into the global coordinates.
5) Recall that a member global stiffness coefficient represents the force at location and direction _______ required, along with other ____________________________________ to cause a ____, while all other global end displacements are restrained and no ______________________________ are applied along the member.
6) The member end forces (stiffness coefficients) that are required to create unit displacements for each DOF are illustrated in Figures 20-25.

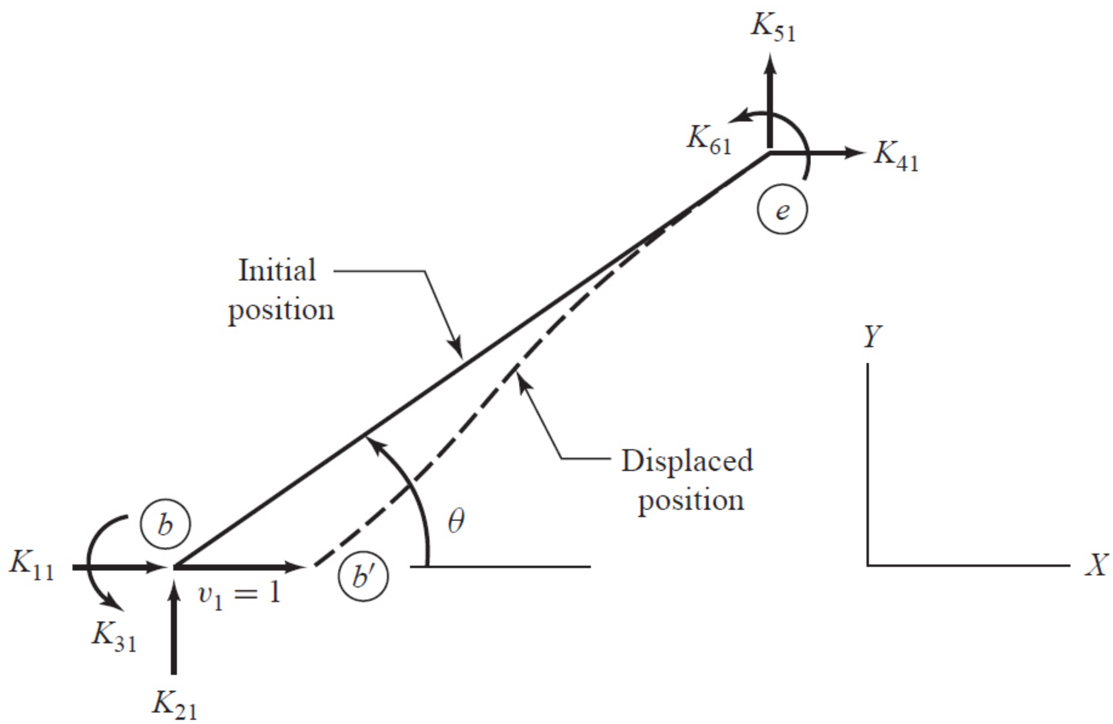


Figure 20. Unit displacement imposed at ____, resulting in ______________________________.

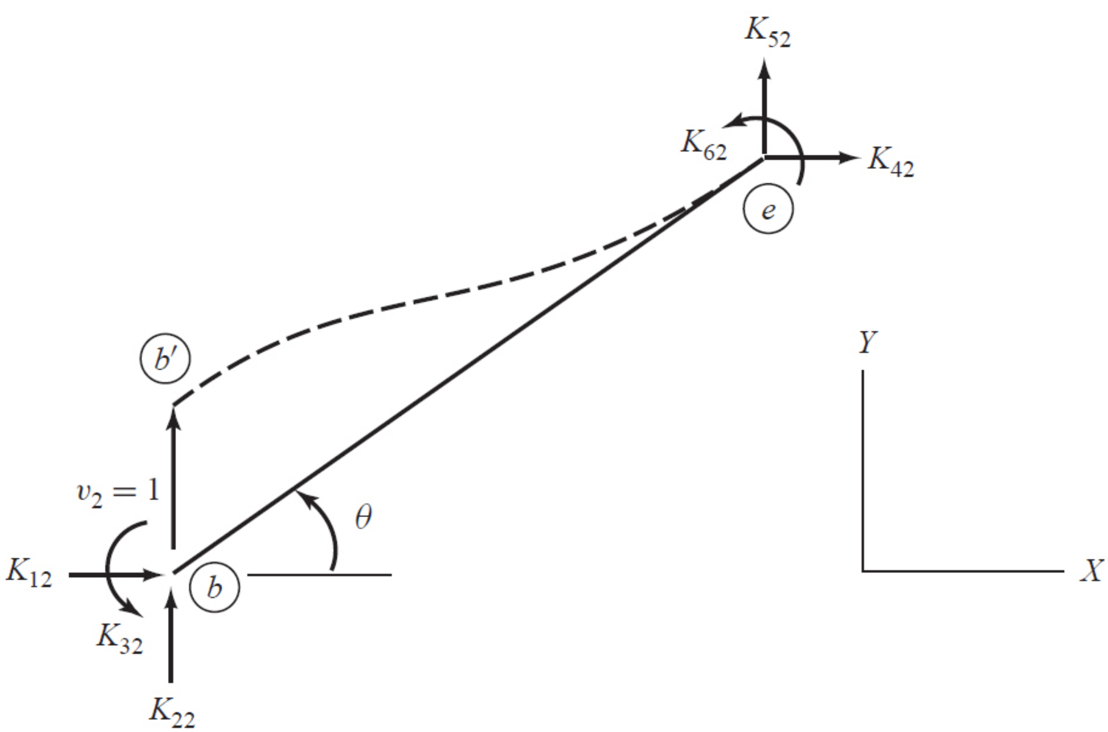


Figure 21. Unit displacement imposed at ____, resulting in ______________________________.

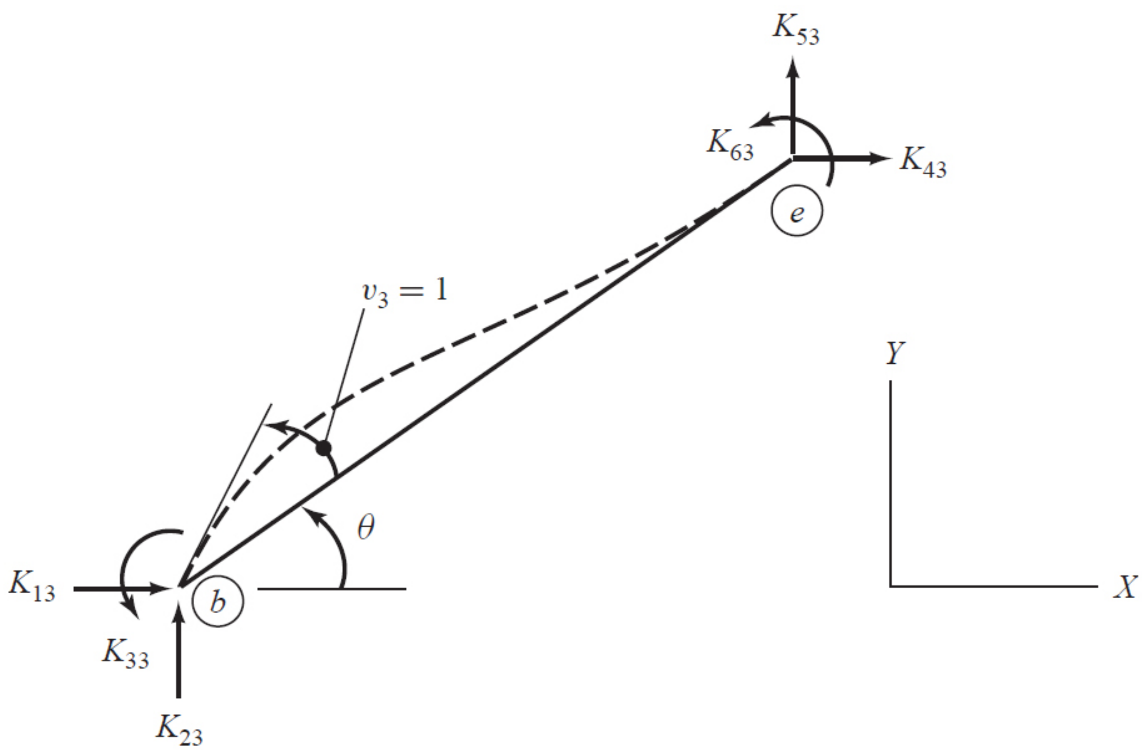


Figure 22. Unit displacement imposed at ____, resulting in ________________________________.

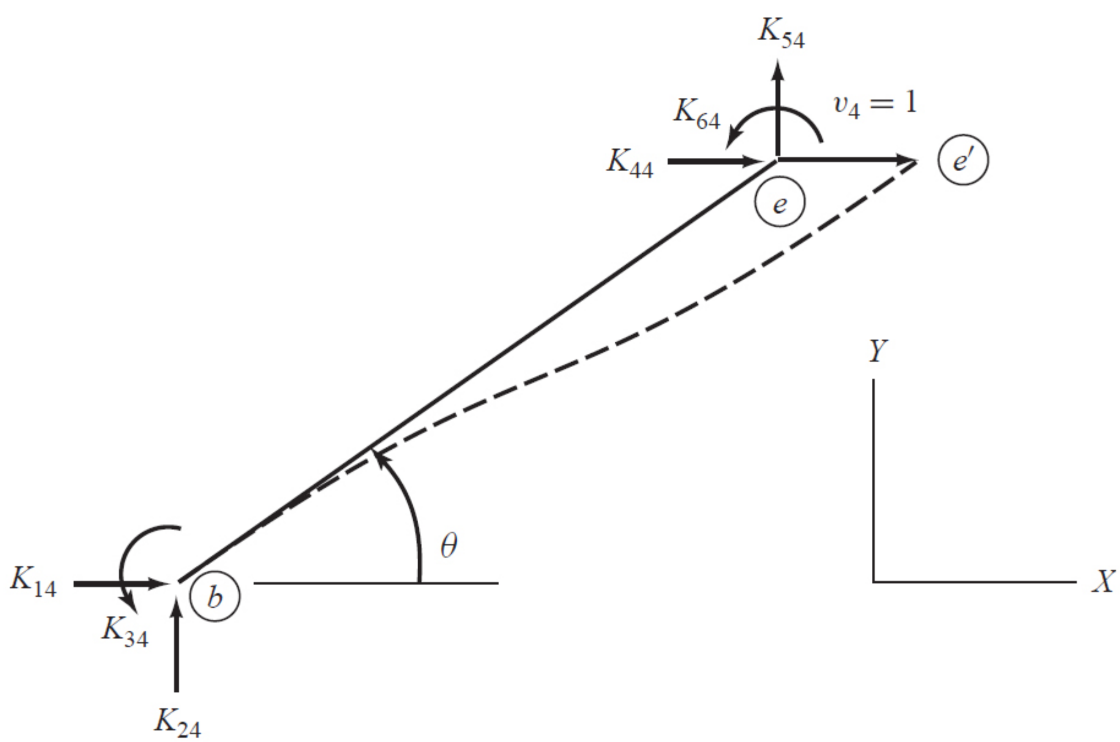


Figure 23. Unit displacement imposed at ____, resulting in ________________________________.

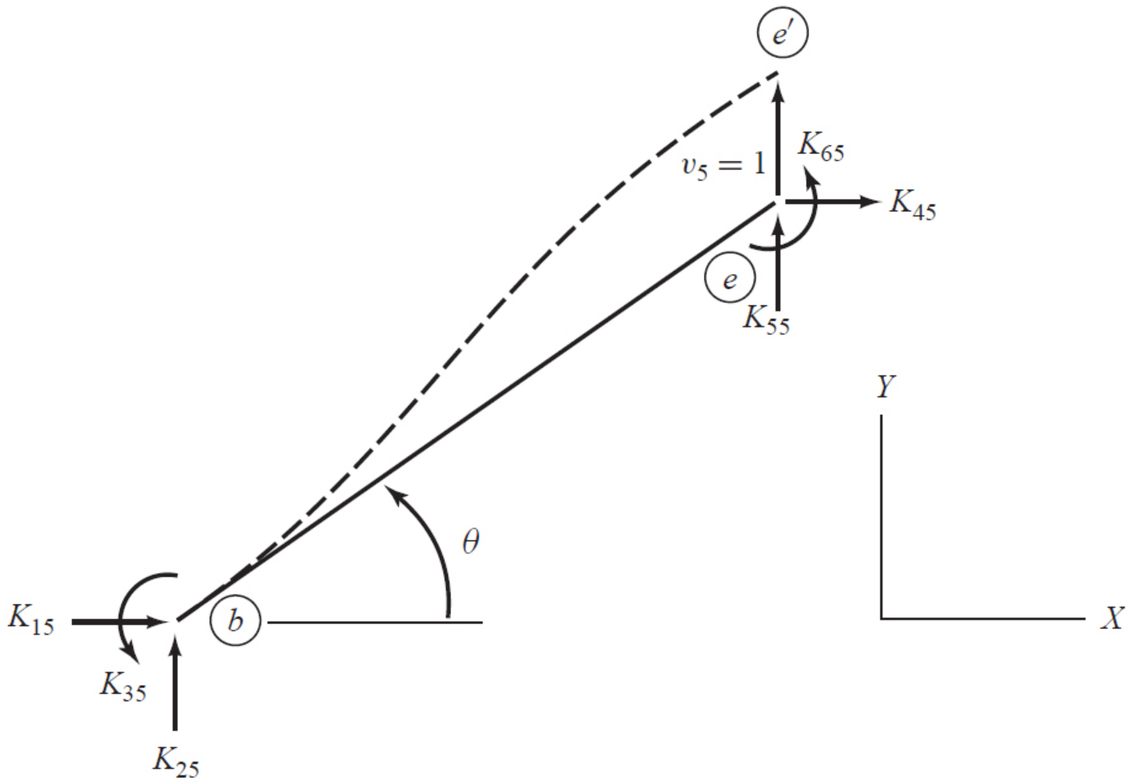


Figure 24. Unit displacement imposed at ____, resulting in ________________________________.

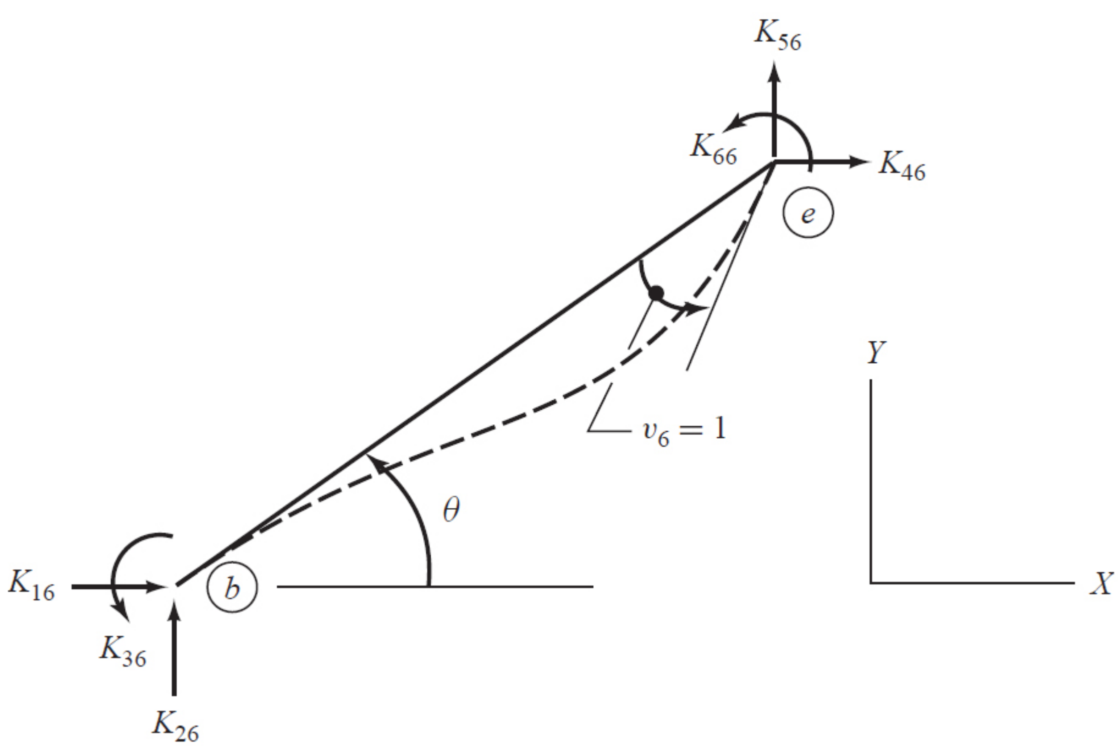


Figure 25. Unit displacement imposed at ____, resulting in ________________________________.

7) To determine the values of first column of [K], one can subject the member m to a unit displacement at ______, while restraining all other ______.
   a. Directly imposing __________, while ________________________________.
   b. Using an algebraic sum of the member end forces in both coordinate systems.

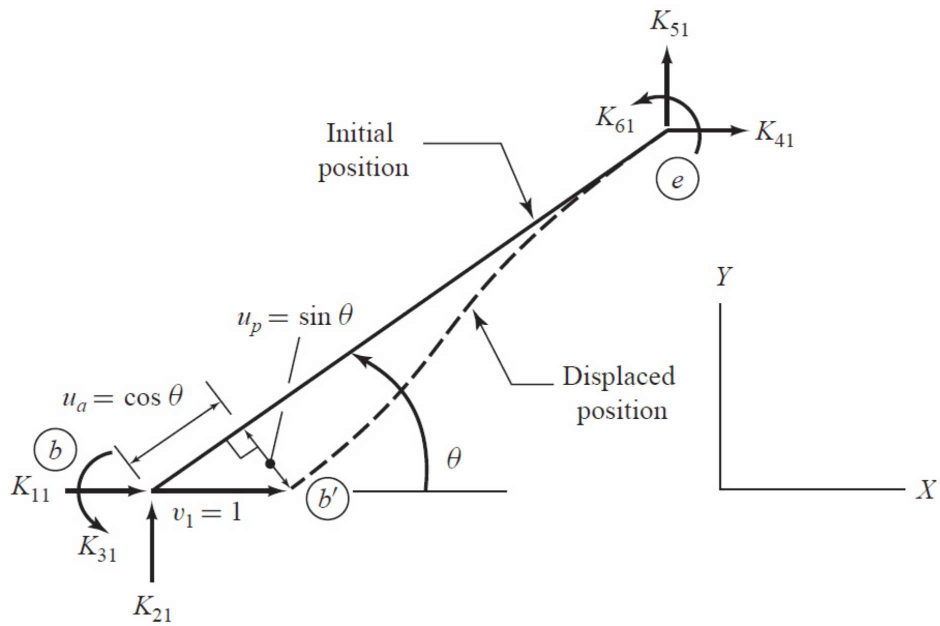


Figure 26. Unit displacement imposed at ____ and its relation in _________________________.

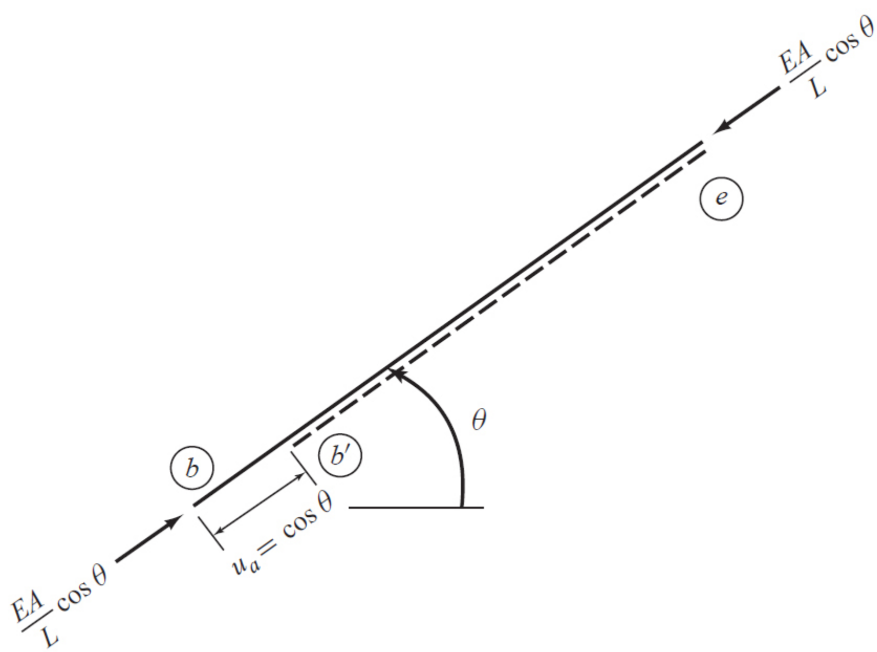


Figure 27. ________ resulting from the release of ______________. Note the developed stiffness coefficients in ____.

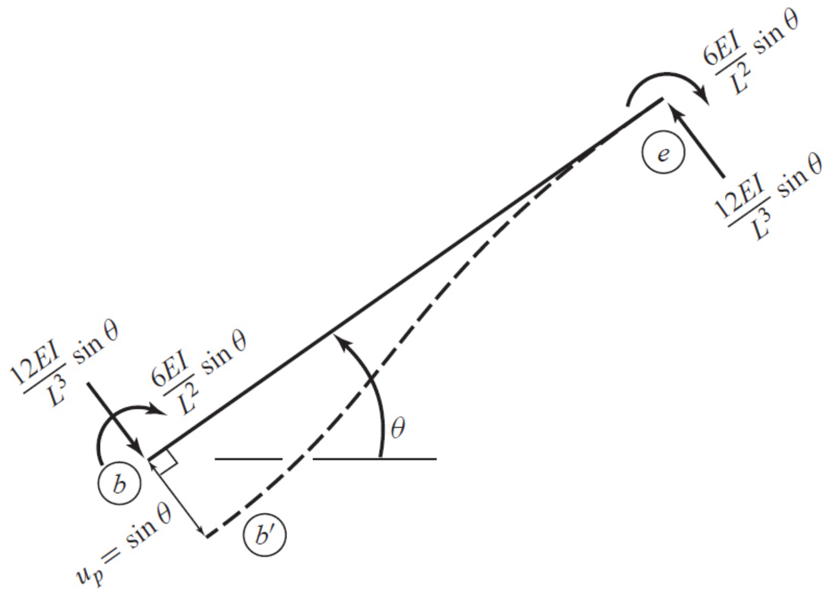


Figure 28. ________ resulting from the release of ______________. Note the developed stiffness coefficients in ____.

8) The remaining columns of [K] can be verified using the same approach.
   a. This means ________ different displacements are imposed in each iteration.

Member Global Fixed-End Force Vector:

1) The relationship of the member global fixed-end force vector (____) and the member local fixed-end force vector can be expressed by:

2) The member global fixed-end forces represent the ________ that develop if the member ends are ____.

3) The orientation of the two fixed-end force vectors are illustrated in Figures 29 and 30.

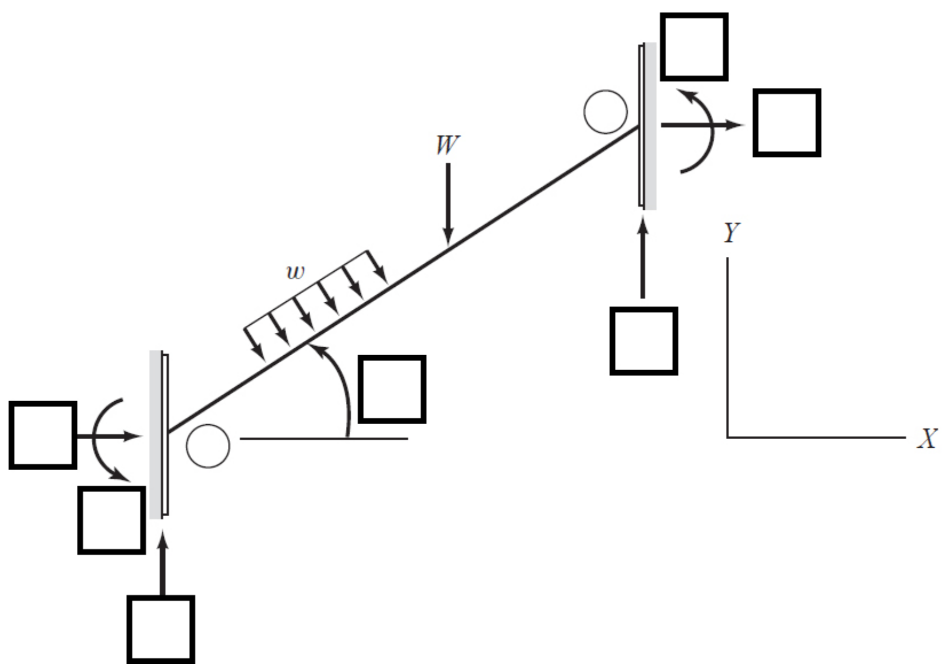


Figure 29. Member fixed-end force vector in ____________ coordinate system.

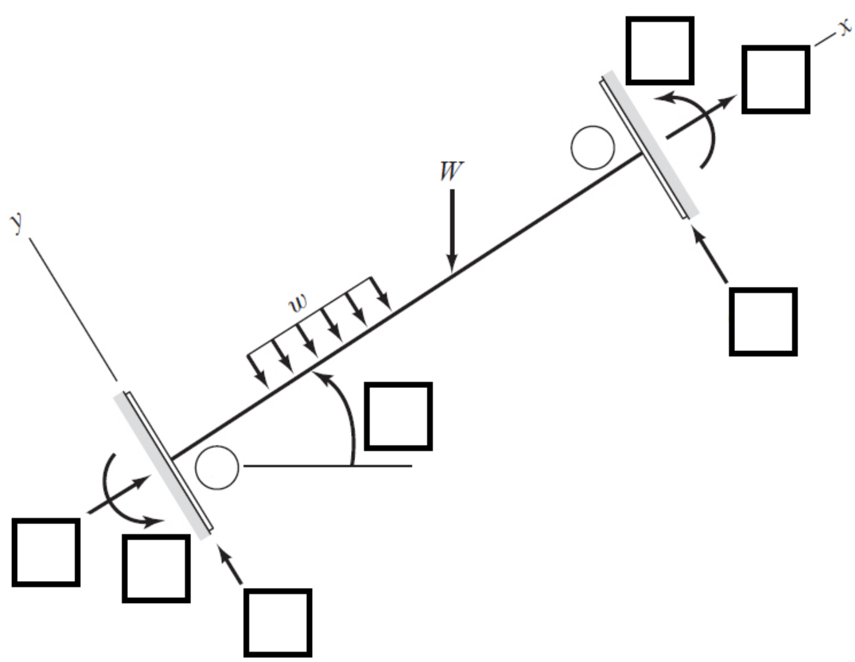


Figure 30. Member fixed-end force vector in ____________ coordinate system.

Structure Stiffness Relations:

1) The process of establishing the structure stiffness (_____) relations for the plane frame is similar to that of beams and trusses.
2) Consider the example plane frame illustrated in Figure 31.
    a. This frame has ____ degrees-of-freedom, as illustrated in Figure 32.
3) One of the more convenient ways to assemble the structure stiffness matrix is using code numbers. Is this the only method? _____________
    a. Another method is by imposing ____
    _____________________________________.
    b. Code numbers can be used to formulate the required force vectors as needed.

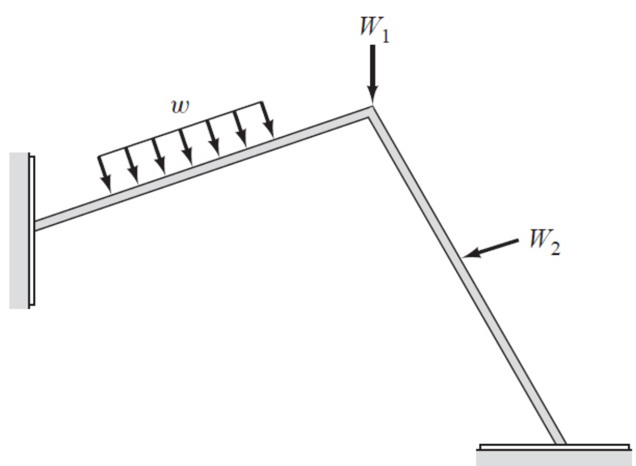


Figure 31.Example plane frame with arbitrary loading at nodes and along member lengths.

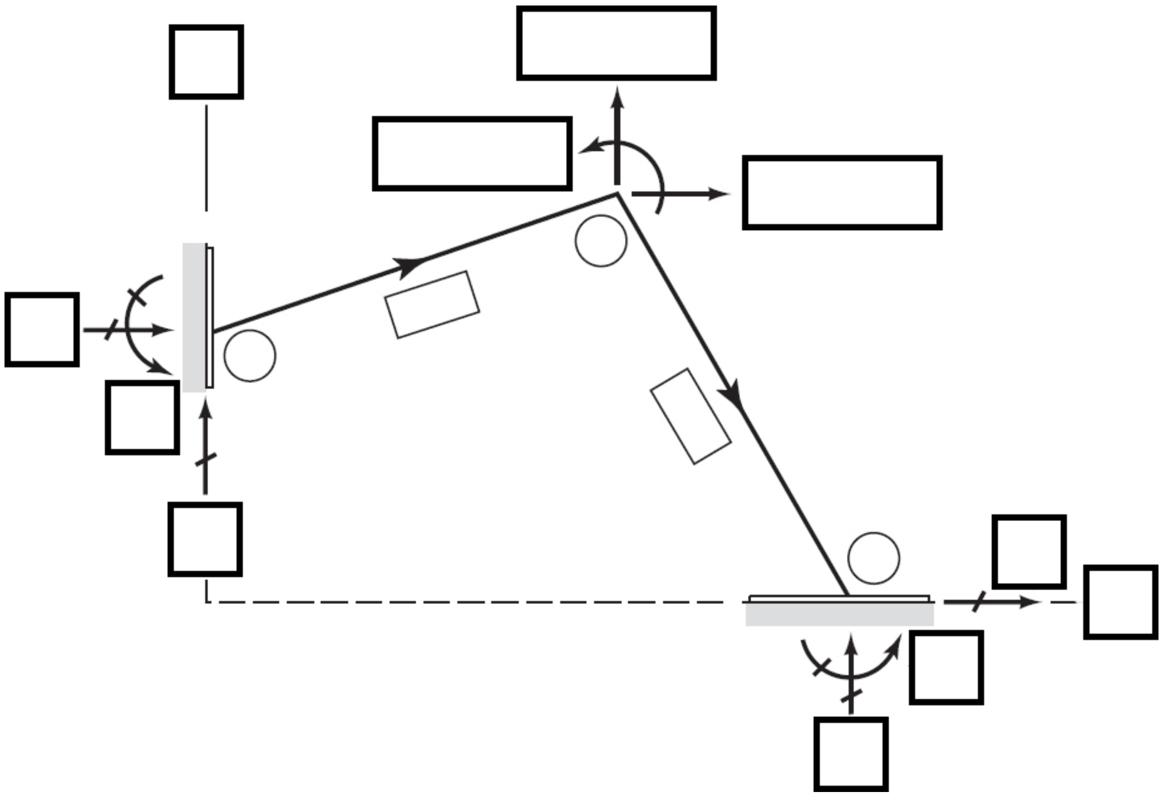


Figure 32. Discretization of example plane frame.

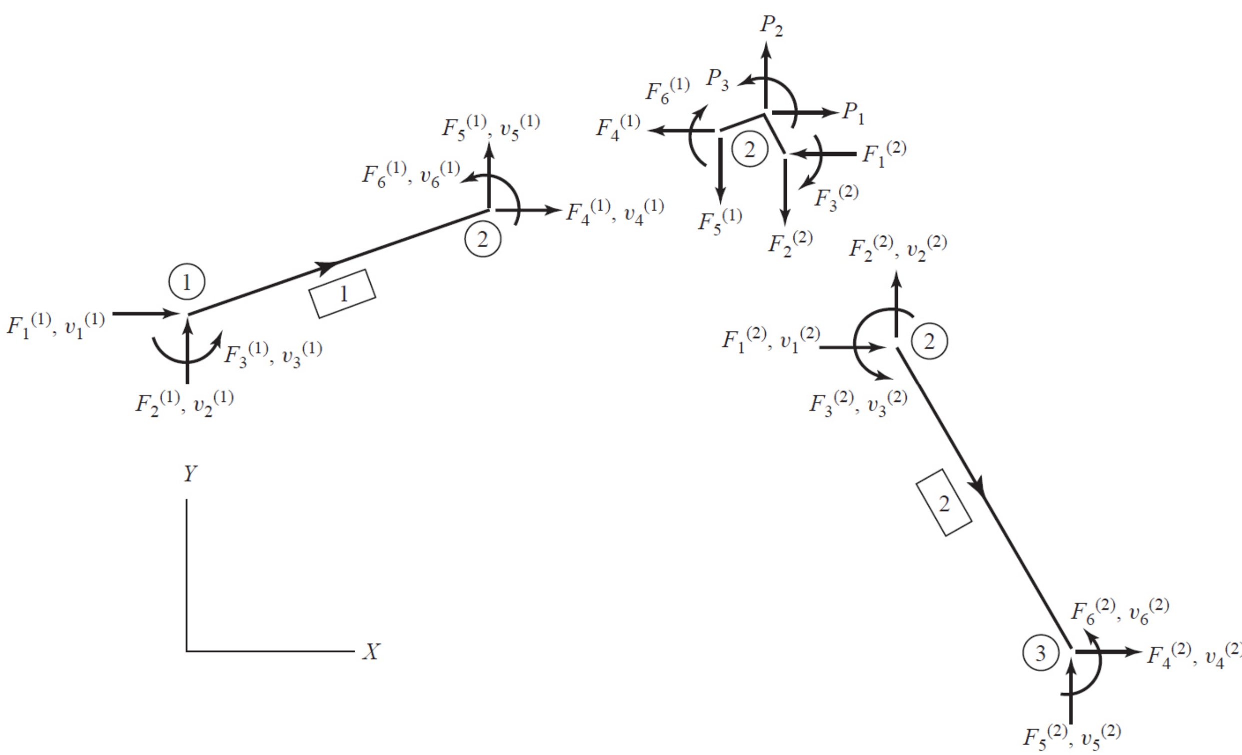


Figure 33. Member end forces for each member of the example plane frame.

General Procedure for Analysis for Frames:

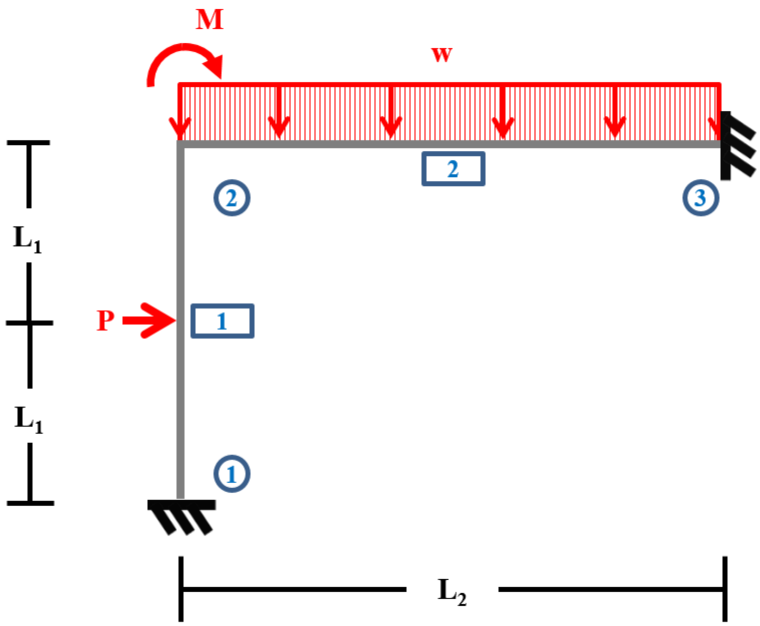
The procedure for frames is very similar to that of plane trusses and beams. The general steps can be summarized as:

1) Discretize an analytical model to represent the structure.
    a. Draw and label line diagram.
    b. Establish the local and global coordinate systems.
    c. Identity the degrees of freedom and the restrained coordinates.
2) Construct the structure stiffness matrix, [S] and fixed-joint force vector [P_f].
    a. Determine each member's global stiffness matrix, [K].
        i. Determine local stiffness matrix, [k].
        ii. Apply the transformation matrix, [T], if appropriate.
    b. Evaluate the numerical values of [Q_f].
    c. Identify the code numbers and assemble the elements of [S] and [P_f].
3) Formulate the joint load vector, [P].
4) Determine the joint displacements (DOFs) using the structure stiffness relationship:
    a. Joint translations are considered positive when ______________________________.
    b. Joint rotations are considered positive when ________________________________.
5) Use the computed joint displacements to find the member end forces, member end displacements, and support reactions as needed.
6) Check equilibrium to verify the solution.

Frames: Example

Example #1

For the steel frame illustrated below, determine the joint displacements, member local end forces, and the support reactions. Use the matrix stiffness method.



Additional Information:
Nodes 1 and 3 are fixed
w = 1.5 kip/feet
P = 40 kip
M = 75 kip-feet
L_1 = 10 feet
L_2 = 30 feet
E = 29,00 ksi
A = 10.3 in^2
I = 510 in^4

Solution:

Identify the number of degrees of freedom. NDOF = ________.

Identify the number of reactions. NR = ________.

E = ______________.

A = ______________.

I = ______________.

Examine member 1.

[k] = f(E,I,A,L); E = 29,000 ksi I = 510 in^4 A = 10.3 in^2 L = 240 in

Code numbers: ___ ___ ___ ___ ___ ___

Local stiffness matrix:

$$[k] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

Evaluation:

$$[k_1] = \begin{bmatrix} 1244.6 & 0 & 0 & -1244.6 & 0 & 0 \\ 0 & 12.839 & 1540.6 & 0 & -12.839 & 1540.6 \\ 0 & 1540.6 & 246500 & 0 & -1540.6 & 123250 \\ -1244.6 & 0 & 0 & 1244.6 & 0 & 0 \\ 0 & -12.839 & -1540.6 & 0 & 12.839 & -1540.6 \\ 0 & 1540.6 & 123250 & 0 & -1540.6 & 246500 \end{bmatrix} (k, in)$$

Fixed End Forces:

$$FA_b = FA_e =$$

$$FS_b = FS_e =$$

$$FM_b = -FM_e =$$

$$\{Q_{f_1}\} = [\qquad\qquad\qquad\qquad\qquad\qquad]^T \ (kips, in)$$

Transformation:

$$\cos(\theta) =$$

$$\sin(\theta) =$$

$$[T] = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 & 0 & 0 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\theta) & \sin(\theta) & 0 \\ 0 & 0 & 0 & -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T_1] = \begin{bmatrix} & & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ 0 & 0 & & 0 & 0 & 0 \\ 0 & 0 & 0 & & & 0 \\ 0 & 0 & 0 & & & 0 \\ 0 & 0 & 0 & 0 & 0 & \end{bmatrix}$$

Global Member Stiffness and Fixed-End Forces:

$$[K] = [T]^T [k][T]$$

$$[K_1] = \begin{bmatrix} 12.839 & 0 & -1540.6 & -12.839 & 0 & -1540.6 \\ & 1244.6 & 0 & 0 & -1244.6 & 0 \\ & & 246500 & 1540.6 & 0 & 123250 \\ & & & 12.839 & 0 & 1540.6 \\ & & & & 1244.6 & 0 \\ & & & & & 246500 \end{bmatrix} \quad (k, in)$$

$$\{F_f\} = [T]^T \{Q_f\}$$

$$\{F_{f_1}\} = [\qquad\qquad\qquad\qquad\qquad\qquad]^T \quad (kips, in)$$

Examining member 2.

[k] = f(E,I,A,L); E = 29,000 ksi I = 510 in^4 A = 10.3 in^2 L = _________

Code numbers: ___ ___ ___ ___ ___ ___

Local stiffness matrix:

$$[k_2] =$$

Fixed End Forces:

$$FA_b = FA_e =$$

$$FS_b = FS_e =$$

$$FM_b = -FM_e =$$

$$\{Q_{f_1}\} = [\qquad\qquad\qquad\qquad\qquad]^T \ (kips, in)$$

Transformation:

$$\cos(\theta) =$$

$$\sin(\theta) =$$

$$[T_2] = \begin{bmatrix} & & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ 0 & 0 & & 0 & 0 & 0 \\ 0 & 0 & 0 & & & 0 \\ 0 & 0 & 0 & & & 0 \\ 0 & 0 & 0 & 0 & 0 & \end{bmatrix}$$

Global Member Stiffness and Fixed-End Forces:

$$[K] = [T]^T[k][T]$$

$$[K_2] =$$

$$\{F_f\} = [T]^T\{Q_f\}$$

$$\{F_{f_1}\} = [\qquad\qquad\qquad\qquad\qquad\qquad]^T \ (kips, in)$$

Assembling the Structure Stiffness Matrix.

NDOF = _______, therefore [S] has the dimensions of ____ x ____

Assembling the Joint Load Vectors (Applied Loads and Fixed-End Forces).

Find the Joint Displacements.

$$\{P\} = [S]\{d\} + \{P_f\}$$

$$\{P\} - \{P_f\} = [S]\{d\}$$

$$\{d\} = [S]^{-1}\left(\{P\} - \{P_f\}\right)$$

$$\{d\} = [S\}^{-1}\left\{\begin{matrix} \quad \\ \quad \\ \quad \end{matrix}\right\}$$

$$\begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} =$$

Compute Member End Displacements and Forces.

Member 1:

$\{v_1\} =$

$\{u_1\} = [T_1]\ \{v_1\} =$

$\{Q_1\} = [k_1]\{u_1\} + \left\{Q_{f_1}\right\} =$

$\{Q_1\} = [k_1]\{u_1\} + \left\{Q_{f_1}\right\} =$

$\{F_1\} = [T_1]^T\ \{Q_1\} =$

Member 2:

$\{v_2\} =$

$\{u_{\mathbf{2}}\} = [T_2]\ \{v_2\} =$

$\{Q_2\} = [k_2]\{u_2\} + \left\{Q_{f_2}\right\} =$

$\{Q_2\} = [k_2]\{u_2\} + \left\{Q_{f_2}\right\} =$

$\{F_2\} = [T_2]^T\ \{Q_2\} =$

Compute the Reactions.

$$\{R\} = \begin{Bmatrix} R_4 \\ R_5 \\ R_6 \\ R_7 \\ R_8 \\ R_9 \end{Bmatrix} =$$