Moment Distribution Method

Lesson Objectives:

1) Identify the formulation and sign conventions associated with the Moment Distribution Method.
2) Derive the Moment Distribution Method equations using mechanics and mathematics.
3) Outline procedure and compute the structural response via Moment Distribution Method.

Background Reading:

1) Read ____

Moment Distribution Method Overview:

1) This method was first introduced in __________ by ________________ for the analysis of ______________________________________.
2) This is another classical formulation of the ________________________.
3) This method only considers ________________________________.
    a. Therefore the assumption is made that ______________________ are negligible.
    b. Reasonable? ______________________________________
4) The moment distribution method is useful in the approach to perform structural analysis:
    a. Gain __________________ into the structural ____________ and __________________.
    b. Does not require to solve a ______________________________.
        i. This is required within the ________________________.
    c. Useful method to check ______________________________.

Moment Distribution Method Basics:

1) In order to solve the structure's system of ______________________________equations simultaneously:
   a. The ____ is examined at a single joint.
   b. This is performed in an ____.
2) A consistent sign convention is established where:
   a. ____are positive when ____.
   b. ____are positive when ____.
3) To perform the _________________________________, one needs to identify how the moment distributes at a ____.
   a. The term ____ is introduced and this is a function of the ___________________________________ of members that frame the joints.
   b. Generally this is written as:

Derivation of Member Stiffness Values:

1) Let's consider two different member fixities, _________________________________ and _________________________________ to derive the member stiffness values.
2) Recall from the slope-deflection notes, a ____ is defined as the moment developed at ___________________________________ for an applied external moment ____.
   a. For pinned-fixed, the ____________________ is:
   b. For pinned-hinged, the ____________________ is:

3) Sketch of example member AB where the ends are ______________________________:

4) Curvature diagram sketch of example member AB:

5) Assuming that the member is __________________(note _____ is constant), the moment area theorems can be applied.

6) Since the rotation at end B is _______________ (___________), the tangential deviation (denoted as ___) is ______________.

7) Writing the second moment area theorem:

8) Solving for ______________, one can find the relationship to the __________________:

9) Since ______ is horizontal, one can apply the first moment area theorem as:

    a. Relates the applied end moment and rotation of the corresponding end.

10) Now, lets consider the second type of member. A sketch of the second example member AB where the ends are _________________________:

11) Curvature diagram sketch of example member AB:

12) From examination of the elastic curve, the rotation at the near end can be written as:

13) Assuming the member is prismatic, the second moment area theorem can be applied as:

14) Now with the two aforementioned relationships noted (equations ____ and ____), let's summarize the bending stiffness values.

15) Let ________ be defined as the bending stiffness of a member.

    a. The ____________________ required at one end of the member to cause a _______ ________________________ at that (same) end).

16) For a ______________________________________ member, this bending stiffness can be expressed as:

17) If ___________________________________ is constant, a newly defined ______________ _______________________ (denoted as _____) is:

18) Likewise for a ____ member:

   a. The bending stiffness is:

   b. The relative stiffness is:

19) To summarize for the two member types (where the _______________________ varies):

   a. The relative stiffness is:

   b. The moment at the far end, _______, is:

   c. The carryover moment is:

Distribution Factors Introduction and Derivation:

1) A distribution factor is defined as the ratio of ________________________________
   of each member to the sum of all the ____
   at that joint.
   a. This is commonly denoted as ___________.
2) Sketch of a simple three-member frame structure with an externally ________________
   _______________________:

3) The free body diagram (FBD) can be drawn as:

4) The free body diagram (FBD) of joint B is:

5) Writing the _____________________ equilibrium equation:

6) Note that for the three members:
   a. Member _____ is ____.
   b. Member _____ is ____.
   c. Member _____ is ____.

7) With previous knowledge of the carryover moment, three expressions for end moments can be written as:

8) Substitution of the three equations above (equation ____,____, and ____) into the equilibrium equation, one can write:

9) Let's introduce the terminology of a rotational stiffness.

   a. This is defined as the _________________ required to cause a _______________ _______________ at the joint of interest.

   b. For the example able, the rotation stiffness of joint B is denoted as: ___________.

10) Some rearrangement of terms will allow for an expression of the unit rotation as:

11) Substitution into the previous equations (equation ____,____, and ____), expressions can be written as:

12) This can be generalized with the definition of a ________________________________.
    a. A distribution factor is defined as the __________________________________ applied to the ________________________________ at end B of member i.
    b. Denoted and written in basic form as:

Fixed-End Moments:

1) Just as used in the ________________________________, expressions for fixed-end are also required for the moment distribution method.
2) Unlike the ________________________________, the effects due to ______________ ________________________ and ___________________________________ must be accounted for using FEM.
3) Example of FEM due to weak foundations/support settlements:

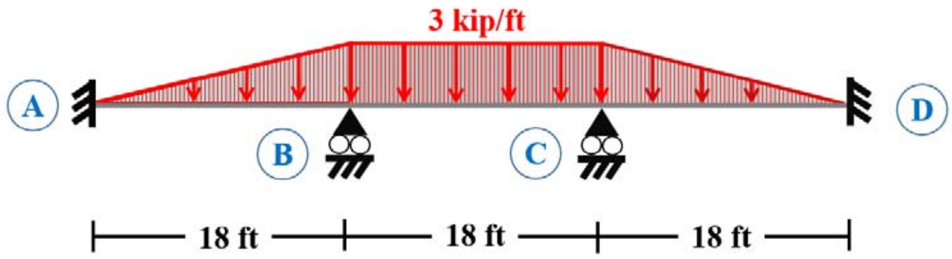
Procedure for Moment Distribution Method:

1) Calculate the distribution factors (______). Check that ________________ at each joint.
2) Compute the fixed end moments. Recall the sign-convention such that ______________ ______________ FEM are established as positive.
3) Balance the moments at each joint that is free to rotate in an iterative approach:
   a. At each joint: first evaluate the ______________________________ and distribute to each member using the __________________________________.
   b. Perform the ____.
   c. Repeat as required until ____.
4) Determine the final member end moments by the sum of ________________________ and ______________________ moments.
   a. Note that moment equilibrium must be satisfied for joints that are ____________ _________________________.
5) Compute the member end shears by ____.
6) Compute the support reactions at joints using ____________________________.
7) Check the calculations of the end shears and support reactions using equilibrium.
8) Draw the shear and bending moment diagrams, if required.

Moment Distribution Method: Example #1

Problem statement:

Determine the member end moments for the three-span continuous beam illustrated below using the moment-distribution method.



Additional information:

Joints A and D are fixed (moment restrained)
Joints B and C are considered as rollers (vertical translation restrained)
$L_{AB} = L_{BC} = L_{CD} = 18\,feet$
$EI = constant\ for\ all\ members$
Note: This beam was previously solved as the first slope-deflection example.

Solution:

1) Determine the distribution factors at joints B and C.

$DF_{BA} =$

$DF_{BC} =$

$DF_{CB} =$

$DF_{CD} =$

Check distribution factors at each joint.
$\Sigma DF_B = DF_{BA} + DF_{BC} =$

$\Sigma DF_C = DF_{CB} + DF_{CD} =$

2) Compute Fixed-End Moments.

$FEM_{AB} =$

$FEM_{BA} =$

$FEM_{BC} =$

$FEM_{CB} =$

$FEM_{CD} =$

$FEM_{DC} =$

3) Balance moments at joints and determine final end moments.
Begin the moment distribution process by balancing joints B and C.

At joint B:

$M_B^{unbalanced} = FEM_{BA} + FEM_{BC} =$

$DM_{BA} = DF_{BA}\left(-M_B^{unbalanced}\right) =$

$DM_{BC} = DF_{BC}\left(-M_B^{unbalanced}\right) =$

At joint C:

$$M_C^{unbalanced} = FEM_{CB} + FEM_{CD} =$$

$$DM_{CB} = DF_{CB}\left(-M_C^{unbalanced}\right) =$$

$$DM_{CD} = DF_{CD}\left(-M_C^{unbalanced}\right) =$$

Perform the carryover moments at the far ends of each beam segment.
Due to the distributed moments at joint B:

$$COM_{AB} = \frac{1}{2}(DM_{BA}) =$$

$$COM_{CB} = \frac{1}{2}(DM_{BC}) =$$

Due to the distributed moments at joint C:

$$COM_{BC} = \frac{1}{2}(DM_{CB}) =$$

$$COM_{DC} = \frac{1}{2}(DM_{CD}) =$$

Now repeat until the unbalanced moments are negligibly small. It is simple to perform this task in a tabular form (reference Table 1):

Table 1. Moment Distribution Table.

Moments	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}
Distribution Factors						
Fixed-End Moments						
Balance Joints						
Carryover						
Balance Joints						
Carryover						
Balance Joints						
Carryover						
Balance Joints						
Carryover						
Balance Joints						
Final Moments						

4) Final end moments are obtained by summing the columns of the moment distribution table (see table above).
5) The member end shears and support reactions are determined via equilibrium.

Special Cases of the Moment Distribution Method:

1) Simple support at one end.
    a. Sketch:

    b. Recall that the bending stiffness is:

    c. At the simple support, the distribution factor is _____.

    d. To apply the moment distribution method, balance the joint only _____________. Leave the free end ___________________, where the moment is ____________.

2) Cantilever overhang at one end.
    a. Sketch:

    b. Section _____ does not contribute to the rotation stiffness at joint ________.
    c. At the free end, the distribution factor is _____.
    d. However, the loads on the cantilever must be still considered at joint _______.

Analysis of Frames with Sidesway:

1) Frame structures will undergo sidesway (____________________ deflection) due to:
    a. ____
    b. ____
    c. ____
    d. ____
    e. ____
2) To analysis a frame that will undergo __________________ using the moment distribution method, solve in two distinct parts:
    a. Frame with ____
    b. Frame with ____
3) Sketch of the actual frame (___________________________):

4) Sketch of the frame with ____ (___________________________):

5) Sketch of the frame with ____
(_____________________________):

6) Sketch of the frame with ____
(_____________________________):

Procedure for Moment Distribution Method for Frames with Sidesway:

1) First solve the frame with external loads for ___________________________________.
    a. Find _____________________________.
2) Find the fictitious reaction (___________________________________) by equilibrium.
3) Analyze a second frame with the fictitious reaction applied in the opposite direction.
4) Superimpose the relationship of:

5) However, one cannot directly compute the values of ______. This requires an in-direct approach.
6) Analyze a third frame structure with an unknown translation (______________), under an externally applied load of unknown magnitude ______.
    a. Note _____ is in the opposite direction of ______.
7) To solve easily, assume that "____" corresponds to an FEM. Find the remaining FEM values and perform the moment distribution method.
    a. Find _____________________________.
8) Find the value of load _____ by equilibrium.
9) The developed moments are linearly proportional to the magnitude of the load.
    a. Therefore find the ratio of:

10) Assemble the frame by superimposing the loads with the equation:

Moment Distribution Method: Example #2 (Frame with Sidesway)

Problem statement:

Determine the member end moments for the frame illustrated below using the moment-distribution method.

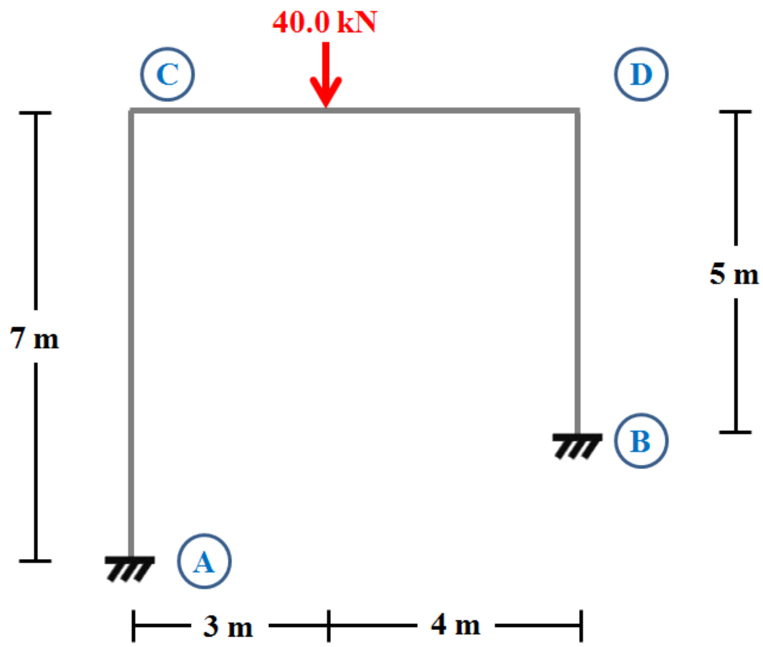


Figure 1. Frame subjected to eccentric loading.

Additional information:

Joints A and B are fixed (moment restrained)

Joints C and CD are considered as rigid

$L_{AC} = L_{CD} = 7\ meters; L_{BD} = 5\ meters$

$EI = constant\ for\ all\ members$

Solution:

1) Determine the distribution factors at joints C and D.

$DF_{CA} =$

$DF_{CD} =$

$DF_{DC} =$

$DF_{DB} =$

Check distribution factors at each joint.

$\Sigma DF_C = DF_{CA} + DF_{CD} =$

$\Sigma DF_D = DF_{DC} + DF_{CD} =$

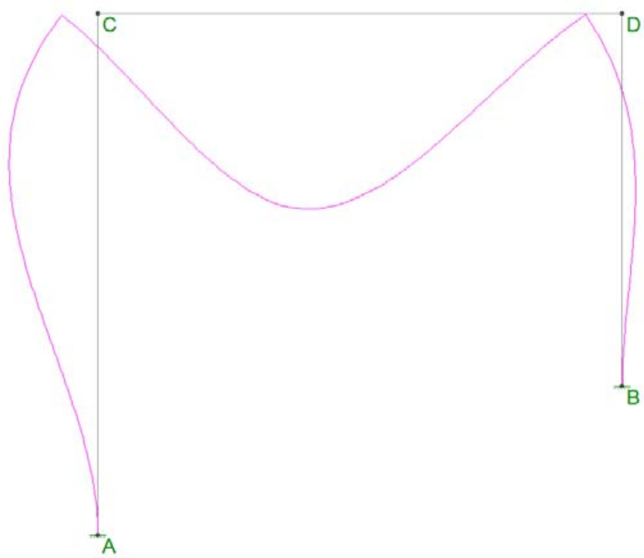


Figure 2. Displaced shape for actual frame with applied loads (from RISA-2D).

2) Frame that is "sidesway prevented":
In the first part of analysis, envision a fictitious roller to prevent ____________________ at joint C. Assume that the joints C and D are therefore clamped against rotation, compute the fixed-end moments due to the external load.

$FEM_{CD} =$

$FEM_{DC} =$

$FEM_{AC} = FEM_{CA} = FEM_{BD} = FEM_{DB} =$

For the case with sidesway translation prevented, perform moment distribution analysis. This procedure is performed below in tabular form (reference Table 2):

Table 2. Moment Distribution Table (**sidesway prevented**).

Moments	M_{AC}	M_{CA}	M_{CD}	M_{DC}	M_{DB}	M_{BD}
Distribution Factors						
Fixed-End Moments						
Balance Joints						
Carryover						
Balance Joints						
Carryover						
Balance Joints						
Carryover						
Balance Joints						
Carryover						
Balance Joints						
Carryover						
Balance Joints						
Carryover						
Balance Joints						
Final Moments						

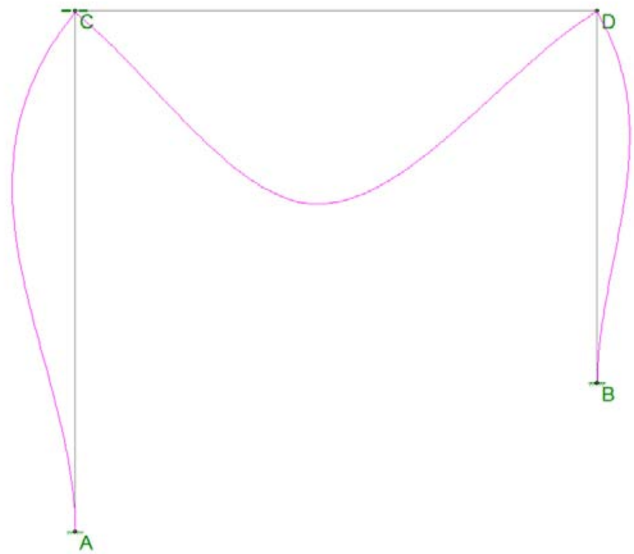
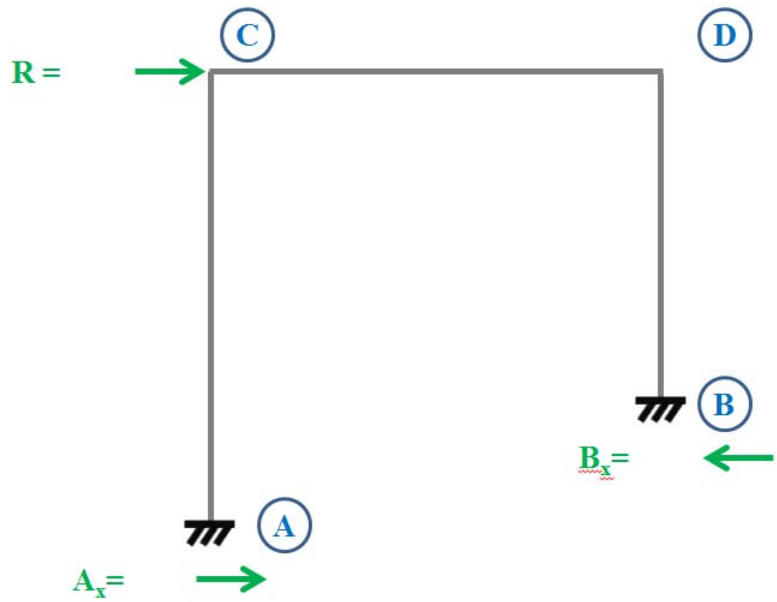


Figure 3. Displaced shape for frame with applied loads where sidesway is prevented (from RISA-2D).

To evaluate the fictitious restraining force *which develops at the imaginary roller at C, calculate the end shears and moments by equilibrium.*

Then consider equilibrium of the entire frame and compute the reaction at C by evaluating the forces in the horizontal direction.



$\Sigma F_x = 0$:

$R_x =$

Note: the computed restraining force acts to the right, indicates that if the roller support was not present, the frame would displace to the left direction.

3) Frame that is "sideway permitted":
The actual frame is not restrained in the lateral direction, therefore one will negate its effect by applying a lateral load to a second frame in the opposite direction.

Recall from the class notes, the moment distribution method cannot directly compute the end moments due to the lateral load. Therefore one will employ an indirect approach where the frame is subjected to an unknown joint displacement, Δ'. Note Δ' is caused by an unknown force, Q, acting at the same location and opposite direction of R (fictitious roller reaction).

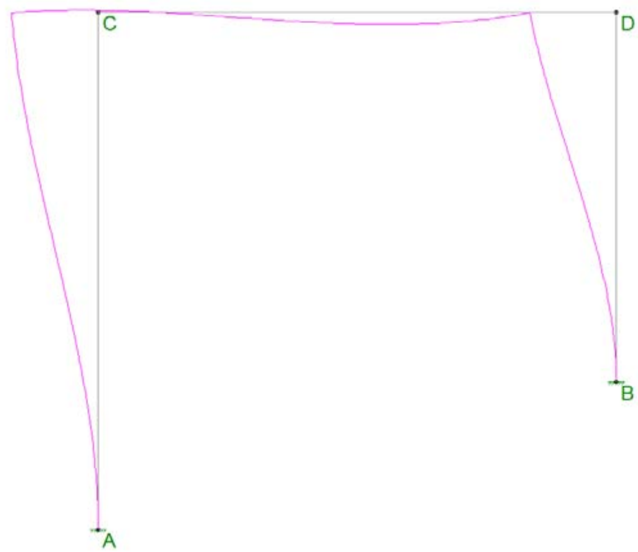


Figure 3. Displaced shape for frame subjected to an arbitrary translation (from RISA-2D).

Assume the joints C and D are clamped against rotation, compute the fixed-end moments due to the translation, Δ'.

$$FEM_{AC} = (-)\frac{6EI\Delta'}{L^2} =$$

$$FEM_{CA} = (-)\frac{6EI\Delta'}{L^2} =$$

$$FEM_{BD} = (-)\frac{6EI\Delta'}{L^2} =$$

$$FEM_{DB} = (-)\frac{6EI\Delta'}{L^2} =$$

$$FEM_{CD} = FEM_{DC} =$$

Negative signs indicate that these moments are ______________________________

In lieu of computing the FEM for the moment distribution method as a function of Δ', arbitrarily assume one fixed end moment.

$FEM_{AC} =$

Solve for Δ' and evaluate values for the remaining fixed end moments using substitution.

$\Delta' =$

$FEM_{AC} = (-)\frac{6EI\Delta'}{L^2} =$

$FEM_{CA} = (-)\frac{6EI\Delta'}{L^2} =$

$FEM_{BD} = (-)\frac{6EI\Delta'}{L^2} =$

$FEM_{DB} = (-)\frac{6EI\Delta'}{L^2} =$

$FEM_{CD} = FEM_{DC} =$

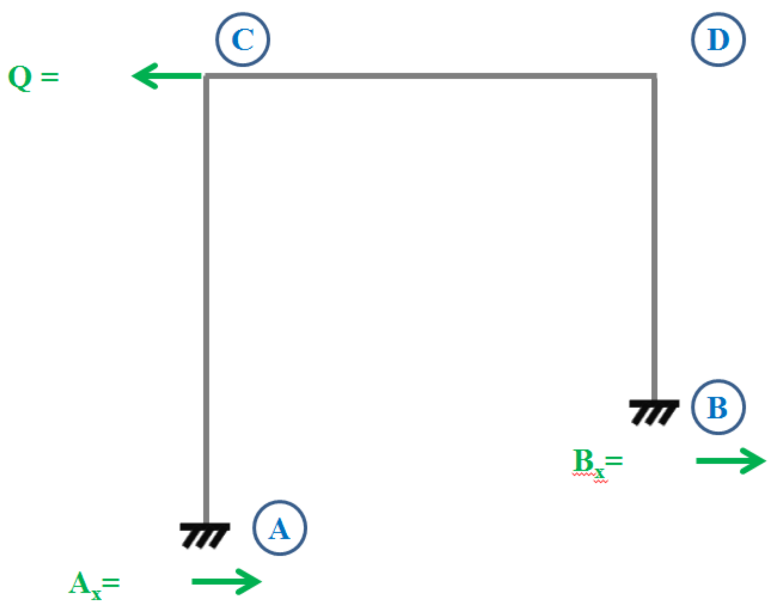
Now the case with frame which undergoes lateral translation is analyzed. Perform moment distribution analysis in the traditional manner, shown here in tabular form (reference Table 3):

Table 3. Moment Distribution Table (**unknown lateral translation due to Q**).

Moments	M_{AC}	M_{CA}	M_{CD}	M_{DC}	M_{DB}	M_{BD}
Distribution Factors						
Fixed-End Moments						
Balance Joints						
Carryover						
Balance Joints						
Carryover						
Balance Joints						
Carryover						
Balance Joints						
Carryover						
Balance Joints						
Carryover						
Balance Joints						
Carryover						
Balance Joints						
Final Moments						

To evaluate the magnitude of Q which resulted in a lateral translation of Δ', calculate the end shears and moments by equilibrium.

Then consider equilibrium of the entire frame and compute the value of Q by evaluating the forces in the horizontal direction.



$\Sigma F_x = 0:$

$Q =$

Therefore the moments calculated in the second moment distribution method are caused by the lateral force Q. Recall that the moments developed are linearly proportional to the to the magnitude of the load, the desired moments corresponding to the fictitious restraint reaction are then multiplied by the ratio of R/Q.

$$\frac{R}{Q} =$$

4) Actual member end moments by superposition.
Now the actual member end moments can be determined by summing the moments computed due to each moment distribution method.

$$M = M_o + M_R$$

$$M_r = \left[\frac{R}{Q}\right] M_q$$

where:
$M_o = moments\ created\ due\ to\ external\ loads\ where\ sidesway\ is\ prevented$
$M_r = moments\ created\ due\ to\ fictitious\ reaction\ (opposite\ direction)$
$M_q = moments\ created\ due\ to\ an\ arbitrary\ translation$

Computing end moments:

$$M_{AC} = M_{o\,AC} + \left[\frac{R}{Q}\right] M_{Q_{AC}} =$$

$$M_{CA} = M_{o\,CA} + \left[\frac{R}{Q}\right] M_{Q_{CA}} =$$

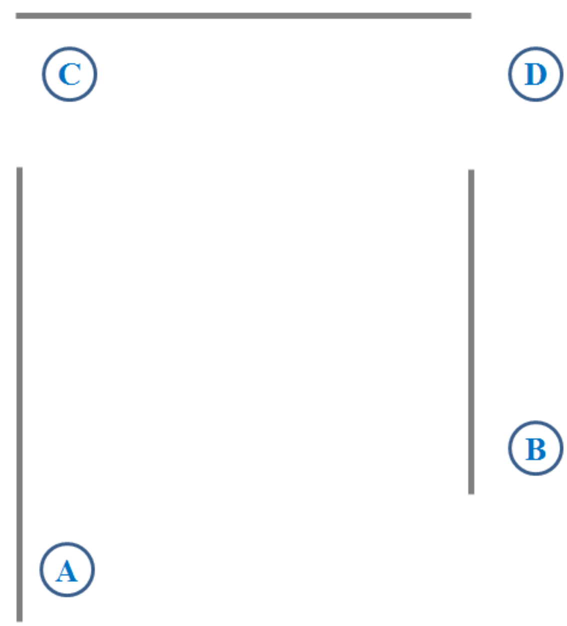
$$M_{CD} = M_{o\,CD} + \left[\frac{R}{Q}\right] M_{Q_{CD}} =$$

$$M_{DC} = M_{o\,DC} + \left[\frac{R}{Q}\right] M_{Q_{DC}} =$$

$$M_{DB} = M_{o\,DB} + \left[\frac{R}{Q}\right] M_{Q_{DB}} =$$

$$M_{BD} = M_{o\,BD} + \left[\frac{R}{Q}\right] M_{Q_{BD}} =$$

5) The actual member end shears and support reactions are determined via equilibrium.



$A_x =$

$B_x =$

$A_y =$

$B_y =$

$M_A =$

$M_B =$