Slope Deflection Method

Lesson Objectives:

1) Identify the formulation and sign conventions associated with the Slope Deflection method.
2) Derive the Slope Deflection Method equations using mechanics and mathematics.
3) Describe the concept of fixed-end moments.
4) Outline procedure and compute the structural response using the Slope Deflection Method.

Background Reading:

1) Read ____

Slope Deflection Method Overview:

1) This method was first introduced in ______________ by ________________________ for the analysis of ____.
2) This is the classical formulation of the ______________________________________.
3) Its formulation extends directly in the ______________________________________ ______________________________ analysis method that will be detailed at a later time.
4) This method only considers ____.
    a. Therefore the assumption is made that ________________________________ are negligible.
    b. Reasonable? ____
5) In this method, the primary unknowns are _______________________________ and they are solved initially using ____.
    a. Afterwards ______________________ and ____________________________ ______________________________.
6) First let's establish fundamental relationships needed to derive the slope deflection method.

Slope Deflection Fundamental Relationships:

1) When the structure of interest, (____________________________ or __________________) is subjected to external loads, ____ develop at the individual member ends.
2) The primary goal of the slope deflection equation is to relate the _____________________ to the ____________________________ and ______________________________ at its ends along with the ____.
3) For derivation, let's consider a beam as illustrated in Figure 1, where _________________ ___ is a cut section from a continuous beam.

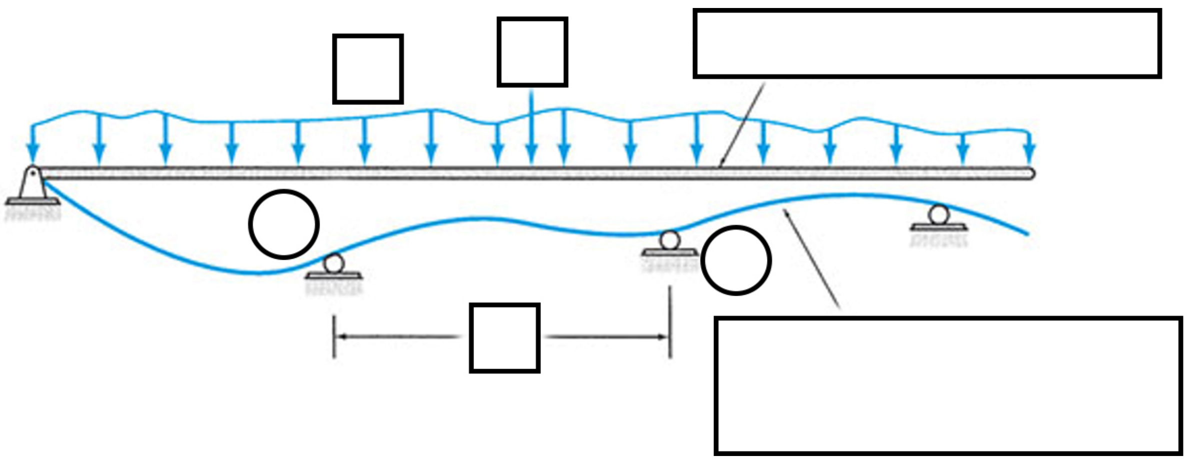

Figure 1. Example continuous beam[1].

4) A consistent sign convention is established where:
   a. ____, _________________ ____________________________, and __________________________________ are positive when ____.
5) The continuous beam section, _______, deforms under _____________________________ ____ (Figure 2).
6) As the member deforms, ____ are induced at the member ends.

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[1] All figures in Slope Deflection were modified from: Kassimali, Aslam. (2014). *Structural Analysis*. 5nd edition. Cengage Learning.

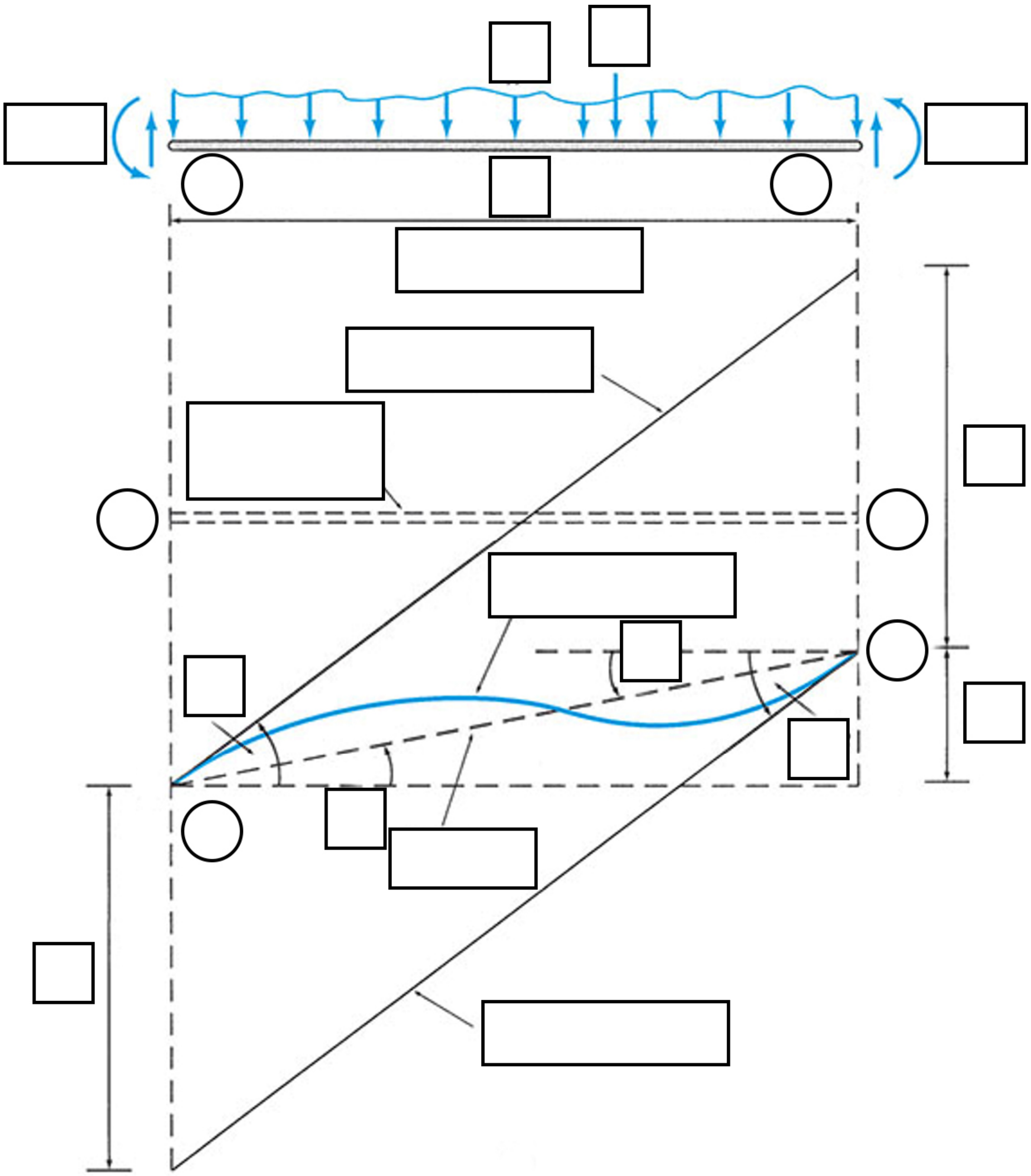


Figure 2. Deformed shape of the continuous beam section.

Notation within the Slope-Deflection Method:

1) The typical notation used within the slope deflection method to denote an end moment:
   a. M_{AB} = moment at the end of _______ of member _________.

   b. Alternatively: A can refer to ___________________, while B denotes the _________________.

2) The rotation at end ____ of a member with respect to the undeformed position is denoted as: _____. This undeformed position for a continuous beam is _____________________.
3) The relative _______________________________ between two ends of the member in a ____ of the undeformed member axis is denoted as: _____.
4) The rotation of the member's chord is denoted as: ____ (pronounced as ________).
   a. A chord is the ____________________________ that connects the ___________ __________________________ due to relative translation (_____).
   b. If small deformations are assumed, geometrically this can be expressed as:

Slope-Deflection Derivation:

1) Derivation of the slope-deflection equations can performed using the ________________ __________________________________.
2) This equation can be written as:

3) From the geometry shown in Figure 2, two equations can be written for the rotation at each end:

4) Substitution in the known relationship of __________

5) Recall that the tangential deviations are specifically detailed as:
   a. ______ denotes the tangential deviation of end _____ from the tangent to the elastic curve at end _____.
   b. ______ denotes the tangential deviation of end _____ from the tangent to the elastic curve at end _____.
6) These tangential deviations can be found using the _________________________ _____________________________.
   a. How? _________________________________.
7) To construct the required bending moment diagram, apply __________, __________, and the ___________________________________ separately over the beam section with ____.
   a. This will create a __________________________ simple bending moment diagram as illustrated in Figure 3.

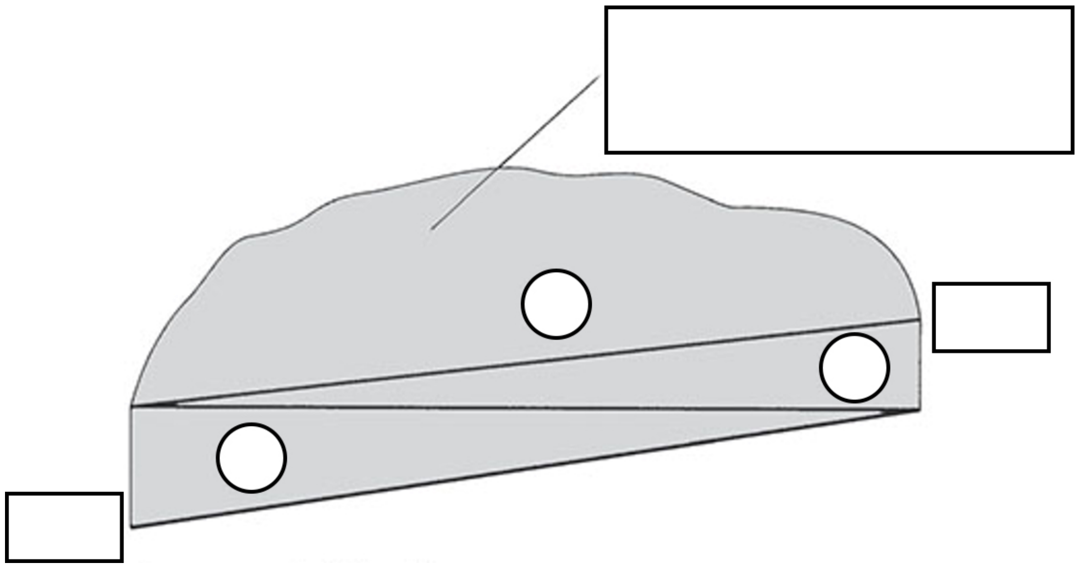


Figure 3. Illustration of the bending moment diagram along the continuous beam section.

8) Within the derivation, another assumption is made that the member section is ____________________________ (where _______ is constant). This allows for a simple application of the ____.
    a. Note the required tangential deviations are illustrated in Figure 4.
9) In Figure 4, ______ and ______ denote moments about the ends ______ and ______, respectively, of the area under the simple beam bending moment diagram.
10) In this same figure, ______ denote an arbitrary __________________________________ ____.

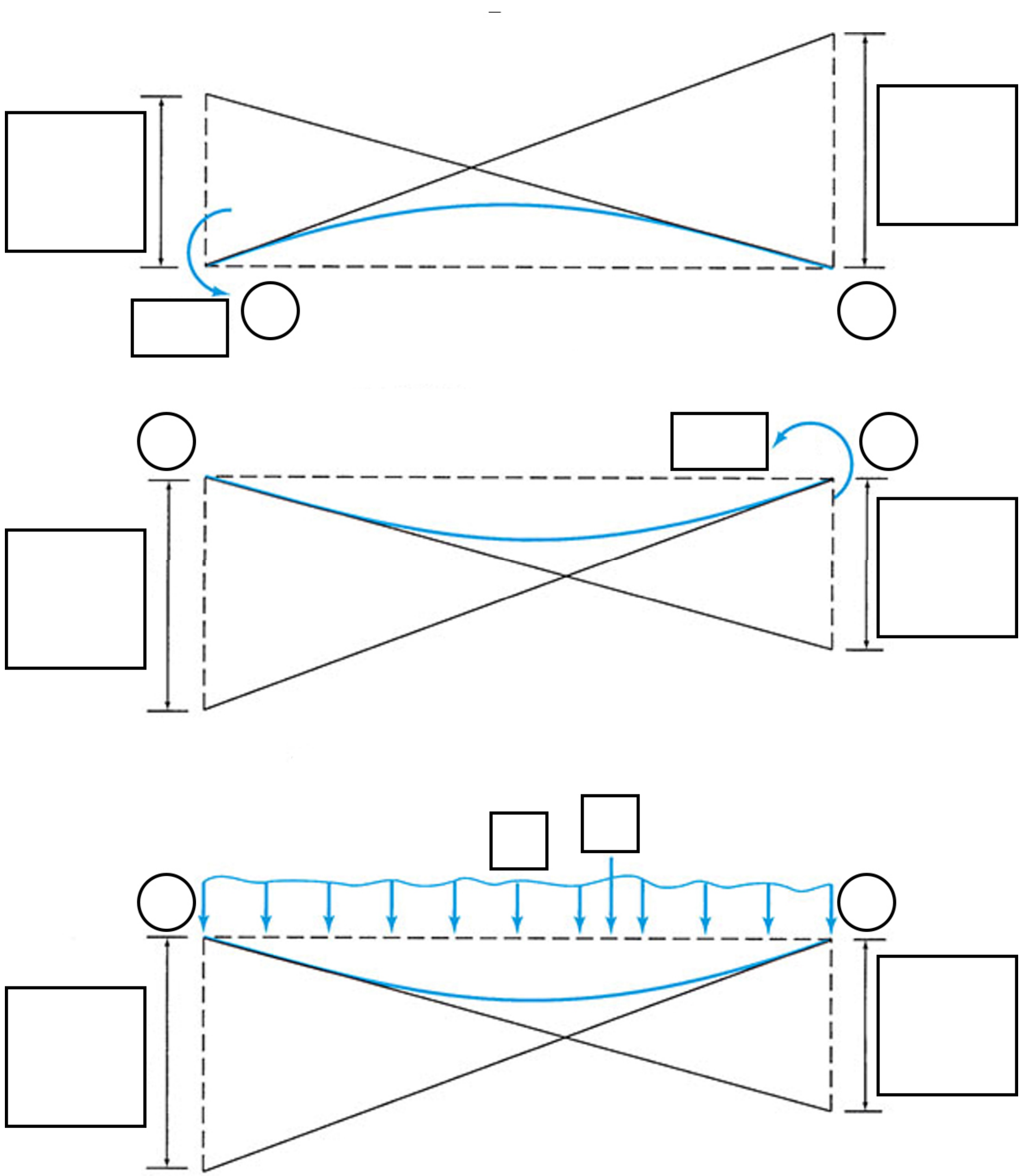


Figure 4. Tangential deviations for various moments.

Slope-Deflection Derivation Continued:

1) Using the second moment area theorem, expressions for each of the tangential deviation can be written as:

2) Note the tangential deviations due to _____, _____, and _______________________ are indicated on Figure 4.
3) In this figure, the negative terms indicate tangential deviations in the ________________ _______________________________ to the elastic curve.
4) Substitution of second set of equations (_____ and _____) into the first set of equations (_____ and _____), produces:

5) To solve these two equations (_____ and _____) simultaneously to compute _____ and _____, it is ideal to rewrite equation ______ as:

6) Substitution of equation _____ into _____, allows one to find _____:

7) Substitution of equation _____ into _____, allows one to find _____:

8) These two previous equations (______ and ______) indicate that the moments that develop at the member ends depend on:
    a. ______________________________ of the member ends.
    b. ______________________________ of the member ends.
    c. ____ between member ends.
9) Now let's assume that the member considered is part of a larger structure, where the isolated beam has _____________________________.
    a. Illustrated in Figure 5.
    b. Note that the ____________________________ and ___________________________ of each end is ____________________________.

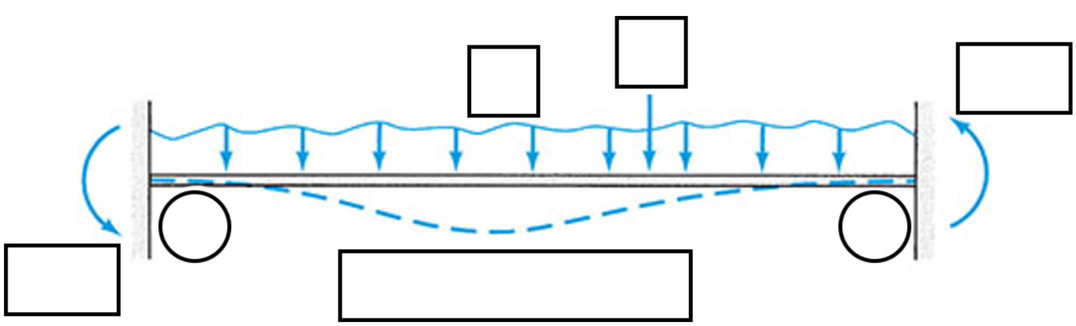


Figure 5. Fixed-end moments (FEM)

10) Using equations _____ and _____ as well as through setting _____, _____, and _____ to _____, expressions can be written for the ___________________________________ as:

11) Where ________________ = the fixed-end moment due to _______________________.

12) These equations (_____ and _____) appear in the previous set after the simultaneous solution was found (equations _____ and _____).

13) Substitution of FEM equation set (_____ and _____) into the previous equation set (_____ and _____):

14) These equations above (_____ and _____) express the moment of the member ends in terms of ______________________ and ______________________ for specified ______________________________.

15) The aforementioned equations (_____ and _____) represent the ____________________ ____. They are only valid when:

    a. The member considered is _____________________________.
    b. The member does not contain a _________________________.
    c. The material behavior is ______________________________.
    d. Deformations are ____________________________________.
    e. Characterize deformation only under __________________________________, therefore _____________________________________ are neglected.

16) This can be written in a single generalized equation as:

Fixed-End Moments:

1) Expressions for fixed-end moments can be derived.
2) However, it is very common to find FEM expressions in a tabular form.
3) Resources typically include:
   a. ______________________________
   b. ______________________________
   c. ______________________________
   d. ______________________________
4) Two example tables (for simplistic externally applied loads) that identify moment diagrams, maximum deflections, and fixed-end moments are included here for reference.

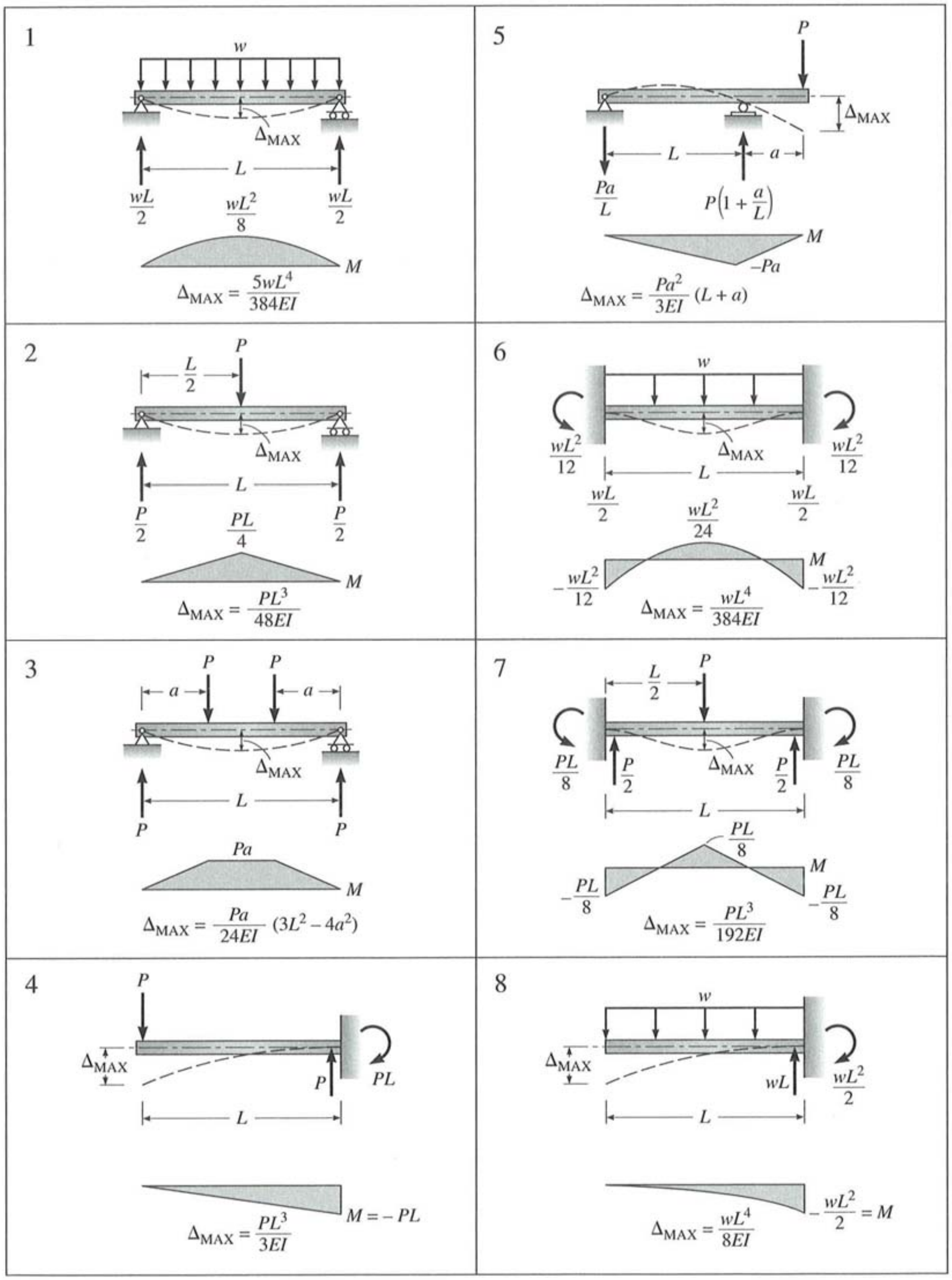


Figure 6. Moment diagrams and equations for the maximum deflection. (Obtained from: Leet, K.M., Uang, C.M., and Gilbert, A.M. (2011) *Fundamentals of Structural Analysis.* 4th Edition. McGraw Hill, New York).

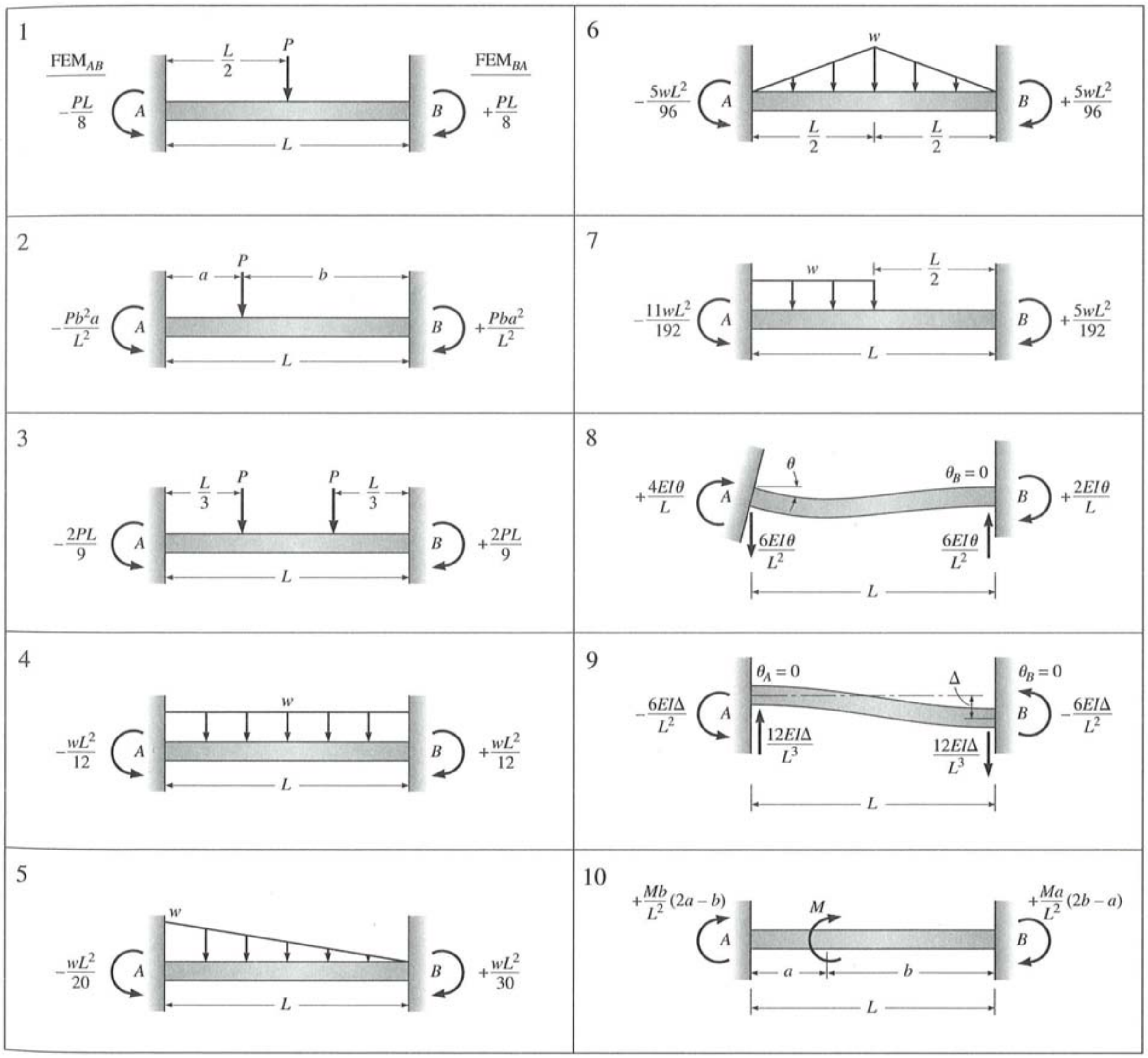


Figure 7. Fixed-End Moments. (Obtained from: Leet, K.M., Uang, C.M., and Gilbert, A.M. (2011) *Fundamentals of Structural Analysis.* 4th Edition. McGraw Hill, New York).

Procedure for Slope-Deflection:

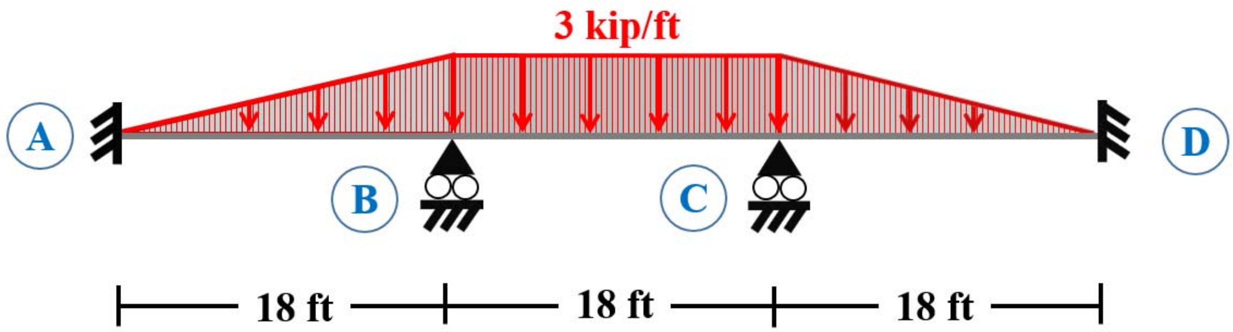
1) Identify the degrees of freedom (DOFs).
    a. For a continuous beam, this is only ___________________.
    b. For a frame structure, this may include __________________ and ___________________.
2) Compute the fixed-end moment (FEM) for each member.
    a. Use handout or other available resources.
3) If support settlement exists, compute the ______________________________________ or ____.
    a. Preferences? ____________________________________
4) Write the slope-deflection equations for each member.

5) Write the equilibrium equations.

6) Determine the deformation at the joints (or member ends) by ______________________.
7) Calculate the member end moments.
8) Check for equilibrium equations.
    a. Equations written in step ____________.
9) Compute the member end shears by _______________________.
10) Compute the support reactions at joints using _____________________________.
11) Check the calculations of the end shears and support reactions using equilibrium.
12) Draw the shear and bending moment diagrams, if required.

Slope Deflection Method: Example #1

Compute the reactions and draw the moment and shear diagrams for the three-span continuous beam as illustrated. Use the slope-deflection method.



1) Identify the degrees of freedom:

2) Calculate the fixed-end moments and chord rotations:

3) Substitute into the slope-deflection equations:
   a. The slope-deflection equation is:

   b. Note the sign conventions of where: ______________________________________, ____________________________, and ________________________________ are positive when ____.

4) Write the equilibrium equations:

5) Solve for the joint deformations:
   a. Substitute slope-deflection equations for M_{BA}, M_{BC}, M_{CB}, and M_{CD} into the equilibrium equations.

6) Compute the member end moments:
   a. Substitute known values of ________ and ________ into the slope-deflection equations.

7) Check equilibrium at joints:

8) Find the member end shears and support reactions using equilibrium and draw the moment and shear diagrams as needed.

Additional Slope-Deflection Insights:

1) From the slope deflection method, a few aspects often reappear in other topics
   a. ____
   b. ____
2) One common structural system is a fixed-pinned beam that is rotated at the pinned end. A sketch is illustrated here:
3) The moment applied at the pinned end is:
4) Therefore the moment at the other end (____________) is:
5) This leads to a key observations, the moment applied at one end __________________ to ______________________________ at the other end.
6) This is known as the carry-over moment:
7) The carry-over moment is a fundamental relationship used within the ______________ ____.

Modified Slope Deflection Equation:

1) Previously the slope deflection equation was only valid if the both of the member ends are ________________________________.
2) A modified equation can be used to account for members that are ________________.
3) Sketch of member:

4) The slope deflection equations can be written and then modified as:

5) Solve for _____ in equation _______ and then substitute into equation _________:

6) Likewise, can do the same derivation for an alternative _____________________ structure:

7) This equation would be written as:

8) Expressing this in a generalized form:

Slope Deflection Method: Example #2

Determine the member end moments and reactions for the frame structure shown below using the slope-deflection method.

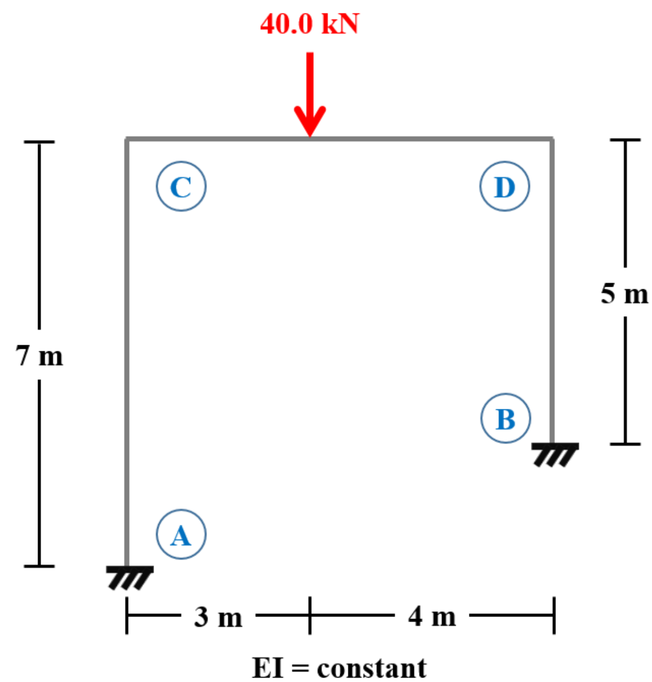


Figure 8. Three member frame for example 2[2].

1) In this structure, the analyst must account for the following:

2) Identify the degrees of freedom:

[2] Example from: Kassimali, Aslam (2014). *Structural Analysis*, 5th Edition. Cengage Learning.

3) Draw the deflected shape (qualitatively):

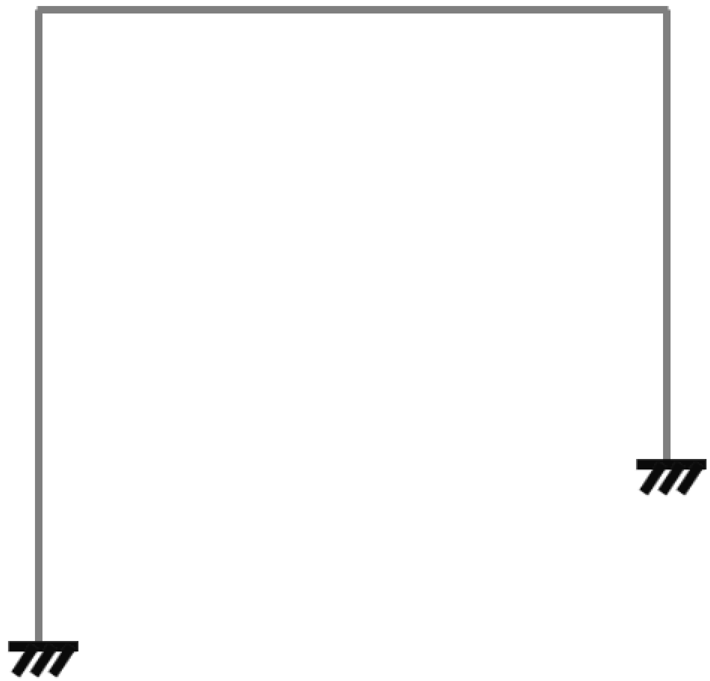


Figure 9. Deflected shaped of frame structure.

4) Calculate the fixed-end moments and chord rotations:

5) Substitute into the slope-deflection equations:

6) Write the equilibrium equations:

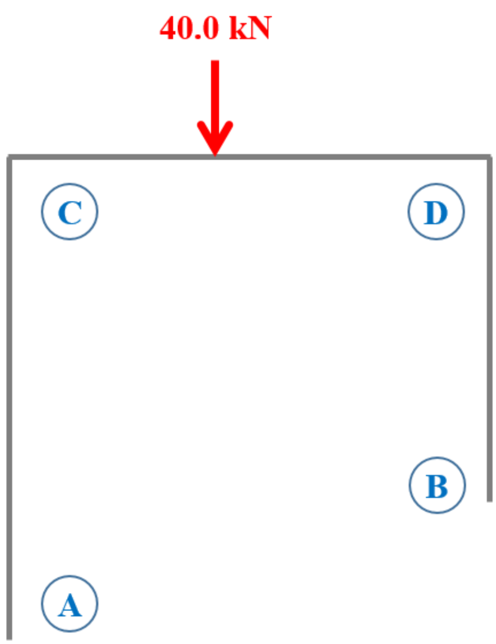


Figure 10. Free body diagram of entire frame structure.

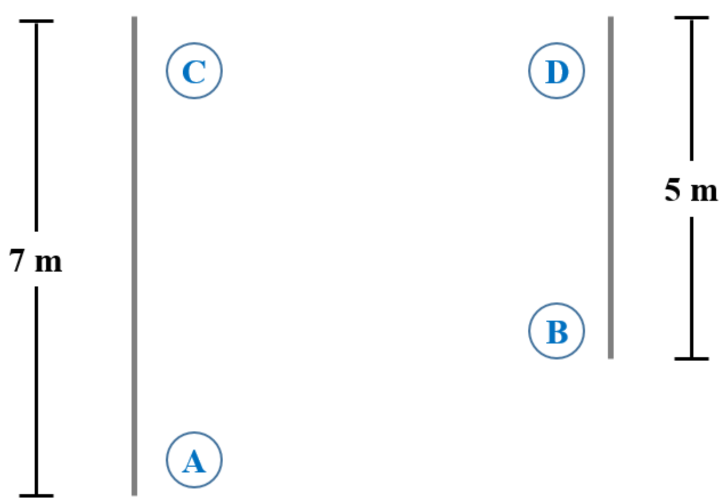


Figure 11. Free body diagrams of the two vertical columns.

7) Solve for the joint deformations:

   a. We now have 3 equations and 3 unknowns, therefore solve the system of equations.

Joint Displacements. To determine the unknown joint displacements θ_C, θ_D, and Δ, we substitute the slope-deflection equations (Eqs. (1) through (6)) into the equilibrium equations (Eqs. (7) through (9)) to obtain

$$1.142EI\theta_C + 0.286EI\theta_D + 0.122EI\Delta = -39.2 \tag{10}$$

$$0.286EI\theta_C + 1.371EI\theta_D + 0.24EI\Delta = 29.4 \tag{11}$$

$$4.285EI\theta_C + 8.4EI\theta_D + 4.58EI\Delta = 0 \tag{12}$$

Solving Eqs. (10) through (12) simultaneously yields

$$EI\theta_C = -40.211 \text{ kN} \cdot \text{m}^2$$

$$EI\theta_D = 34.24 \text{ kN} \cdot \text{m}^2$$

$$EI\Delta = -25.177 \text{ kN} \cdot \text{m}^3$$

8) Compute the member end moments and complete the problem:

Member End Moments. By substituting the numerical values of $EI\theta_C$, $EI\theta_D$, and $EI\Delta$ into the slope-deflection equations (Eqs. (1) through (6)), we obtain

$$M_{AC} = -14.6 \text{ kN} \cdot \text{m} \quad \text{or} \quad 14.6 \text{ kN} \cdot \text{m} \circlearrowright \qquad \textbf{Ans.}$$

$$M_{CA} = -26 \text{ kN} \cdot \text{m} \quad \text{or} \quad 26 \text{ kN} \cdot \text{m} \circlearrowright \qquad \textbf{Ans.}$$

$$M_{BD} = 7.7 \text{ kN} \cdot \text{m} \circlearrowleft \qquad \textbf{Ans.}$$

$$M_{DB} = 21.3 \text{ kN} \cdot \text{m} \circlearrowleft \qquad \textbf{Ans.}$$

$$M_{CD} = 26 \text{ kN} \cdot \text{m} \circlearrowleft \qquad \textbf{Ans.}$$

$$M_{DC} = -21.3 \text{ kN} \cdot \text{m} \quad \text{or} \quad 21.3 \text{ kN} \cdot \text{m} \circlearrowright \qquad \textbf{Ans.}$$

To check that the solution of the simultaneous equations (Eqs. (10) through (12)) has been carried out correctly, we substitute the numerical values of member end moments back into the equilibrium equations (Eqs. (7) through (9)):

$$M_{CA} + M_{CD} = -26 + 26 = 0 \qquad \textbf{Checks}$$

$$M_{DB} + M_{DC} = 21.3 - 21.3 = 0 \qquad \textbf{Checks}$$

$$5(M_{AC} + M_{CA}) + 7(M_{BD} + M_{DB}) = 5(-14.6 - 26) + 7(7.7 + 21.3) = 0 \qquad \textbf{Checks}$$

Member End Shears. The member end shears, obtained by considering the equilibrium of each member, are shown in Fig. 15.17(e).

Member Axial Forces. With end shears known, member axial forces can now be evaluated by considering the equilibrium of joints C and D. The axial forces thus obtained are shown in Fig. 15.17(e).

Support Reactions. See Fig. 15.17(f). **Ans.**

Equilibrium Check. The equilibrium equations check.

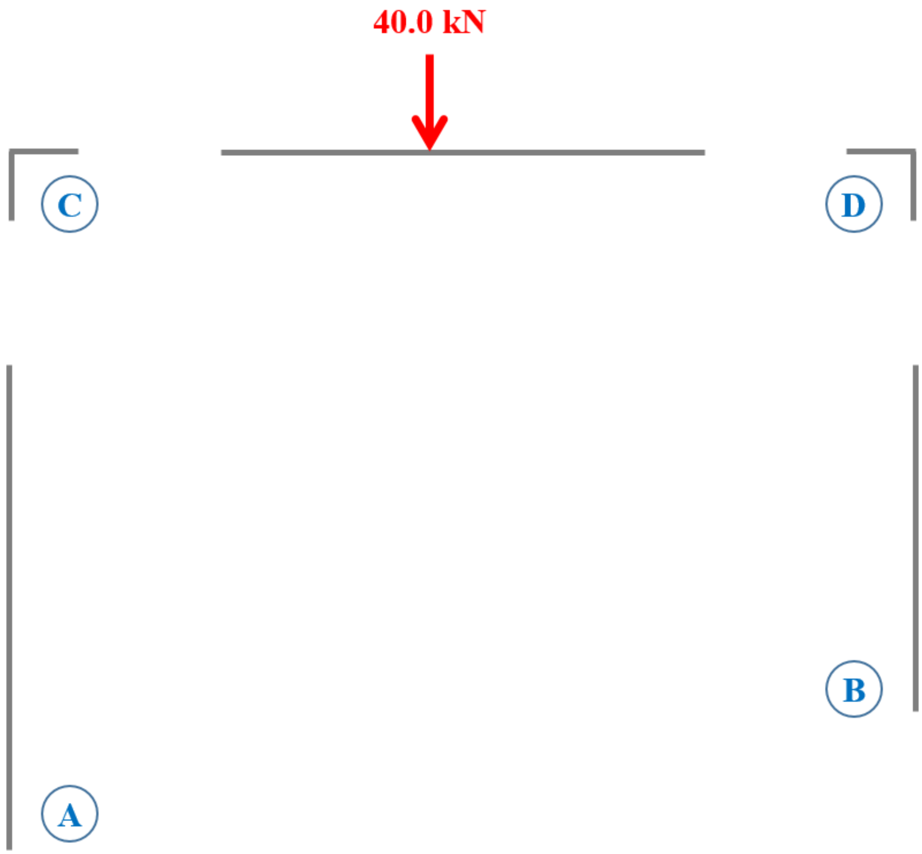


Figure 12. Frame structure: member end moments, shears, and axial forces.

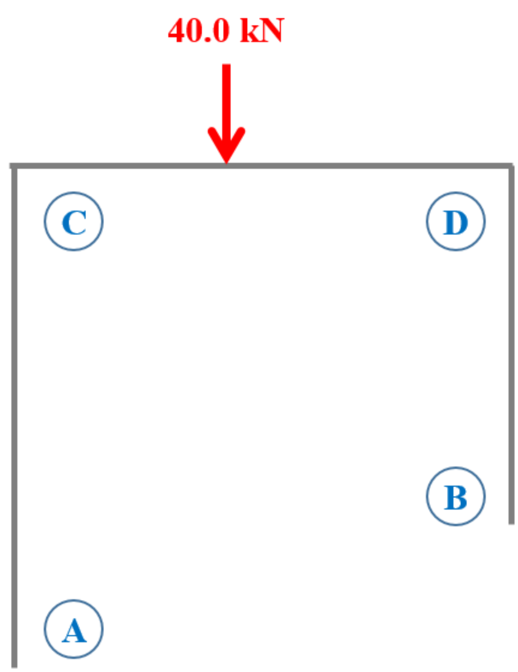


Figure 13. Frame structure: support reactions.