# Additional Topics I: Non-Mechanical Loading

## Lesson Objectives:

1) Derive the methodology to analyze structures for various member releases and secondary effects.
2) Derive the member local stiffness modifications to account for member releases.
3) Compute the structure fixed-joint force vector to account for support settlements.
4) Compute the member fixed-end force vector to account for temperature changes and fabrication errors.

## Background Reading:

1) Read _Kassimali – Chapter 7 (focus on 7.5)________________________

## Introduction:

1) As highlighted for support settlements, ______________________________ and ______________________ can induce significant ____________________ in statically indeterminate structures.
2) However these non-mechanical load sources typically defined relative to ____________ ______________________.
   a. Therefore the ______________________ must be accounted in the _____ ________________________________________.
3) To illustrate, let's examine temperature and fabrication errors separately.

## Temperature Change:

1) One can develop the desired relationships (due to temperature changes) by:
   a. First determining the ______________________ induced by change in temperature at the ends of the member that are free to __________________.

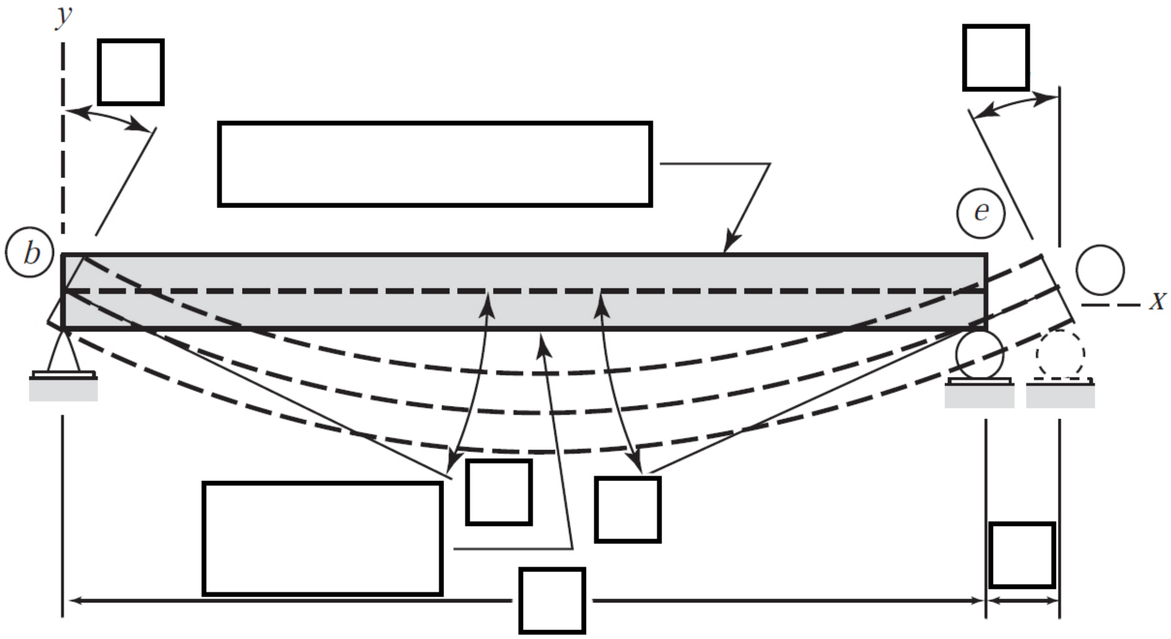
   b. Then the forces required to ____________________________ these member end displacements can be obtained using ____________________________________.

2) Let's consider an example simply support frame member as illustrated in Figure 1.
3) The member is subjected to a uniform heat source at the bottom (or top) surface.
   a. As a result the temperature variation is assumed to be ______________________, where for a symmetric cross-section:

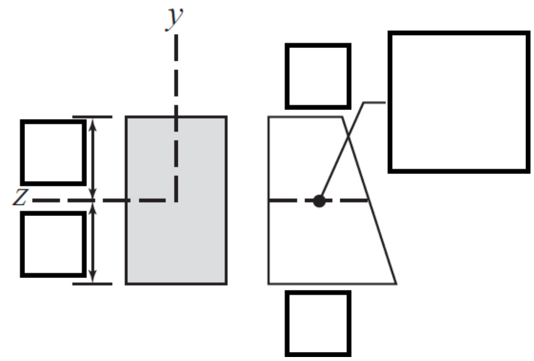
   b. ________ denotes the temperature change at the member _________________.
   c. ________ denotes the temperature change at the member _________________.
4) The simply supported beam from a ______________ (_____________________________) is free to deform in ______________________________________________________.
5) The temperature increase causes the simply supported beam to deform ______________ ____________________________ by the amount _______________ is:

   a. Where ______ is defined as the coefficient of thermal expansion.
6) For the simply supported beam, the other possible deformation is ____________________ ______________________________.
7) If ____________________________, then the member will bend ____________________ ____________________________ causing the ends b and e to rotate _________________.

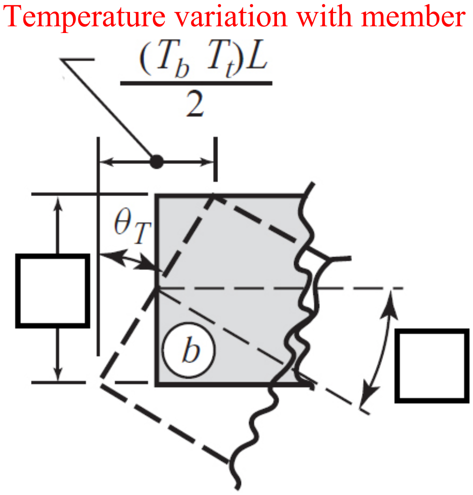
  




(a) Overview of member.




(b) Temperature variation with member depth.

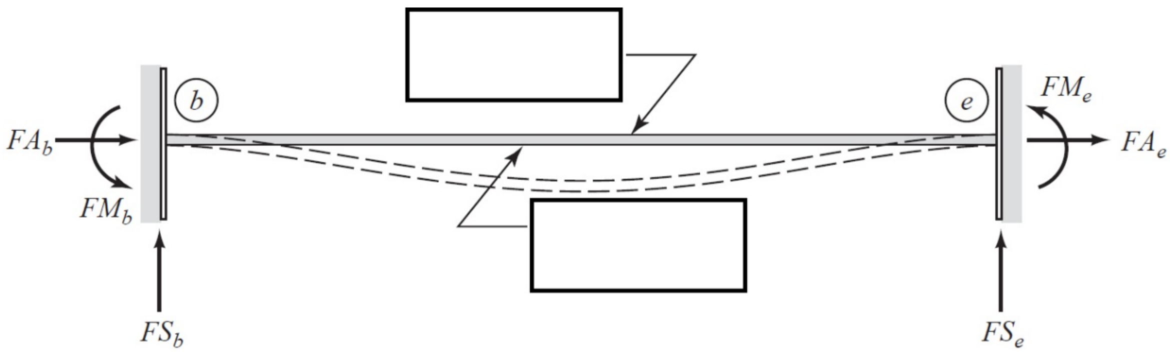



(c) Member end rotation.

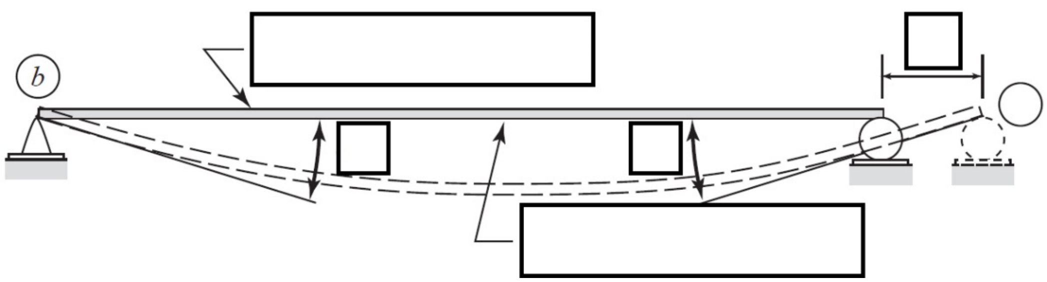
Figure 1. Example of simply supported beam subjected to temperature changes[1].

[1] All figures in Additional Topics I modified from: Kassimali, Aslam. (2012). *Matrix Analysis of Structures*. 2nd edition. Cengage Learning.

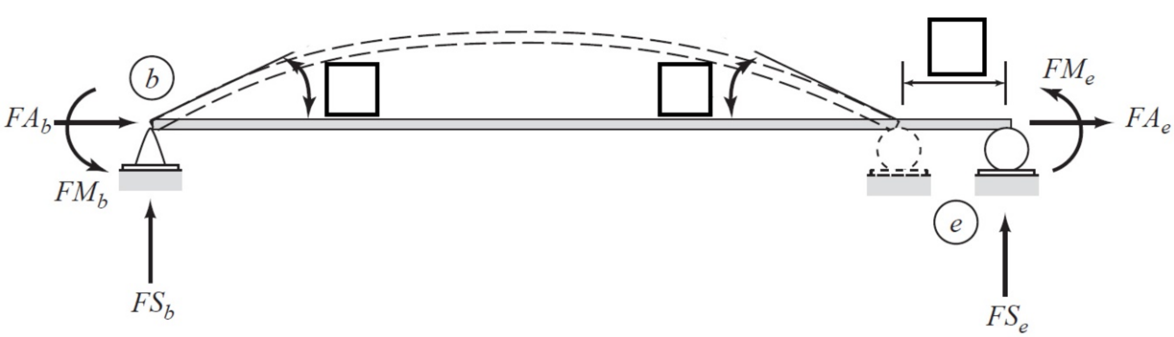
8) Due to the uniform temperature distribution, the ______________________________ at each end are equal in magnitude.
    a. Thus the member end ______________________________ can be related by the temperature change over ______________________________ the difference the elongations of the top and bottom fibers of the member, by its _________________:
    b. Where _______ is the magnitude of the rotations of the member end cross sections.
        i. While also representing the ____________ of the elastic curve at the member ends.
9) Now using the above two derived equations, the local end displacement vector for the simply supported beam due to temperature change is:
    a. Where the sign convention dictates that _______ is negative because it is it ______________________________________________________________________.
10) The ________________________________ required to suppress these end displacements can be found with the ________________________________________________________.
    a. Figure 2a illustrates the fixed-end member subjected to the temperature change and the ______________________________________________________________.
    b. Figure 2b illustrates the first part where the temperature change exists and the ____________________________________________________________ and _______ ____________________________________________________________.
    c. Figure 2c illustrates the second part where the fixed-end forces exists and the ____________________________________________________________.




(a) Fixed member subjected to temperature change and fixed-end forces.




(b) Simply supported beam subjected to temperature change only.




(c) Simply supported beam subjected to fixed-end forces only.

Figure 2. Example fixed beam member subjected to temperature changes.

11) The forces that create the end displacements (________) in Figure 2b can be easily found by multiplying the negative of the member local end displacement vector due to temperature change and the member local stiffness matrix. This is illustrated as:

$$\begin{bmatrix} FA_b \\ FS_b \\ FM_b \\ FA_e \\ FS_e \\ FM_e \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

12) This results in:

$$\begin{bmatrix} FA_b \\ FS_b \\ FM_b \\ FA_e \\ FS_e \\ FM_e \end{bmatrix} =$$

13) Therefore the fixed-end forces for the member of plane frames can be written as:

14) For a beam member, this simplifies to:

15) For a truss member, this simplifies to:

16) The equations above hold true for a ____________________________________________

____________________________________________________.

17) If the temperature change is uniform over the depth, then _______ = _______ = _______, and the equations simplify to:

a. Note only ____________________ exist, as the ___________________________ would be 0.

## Fabrication Error:

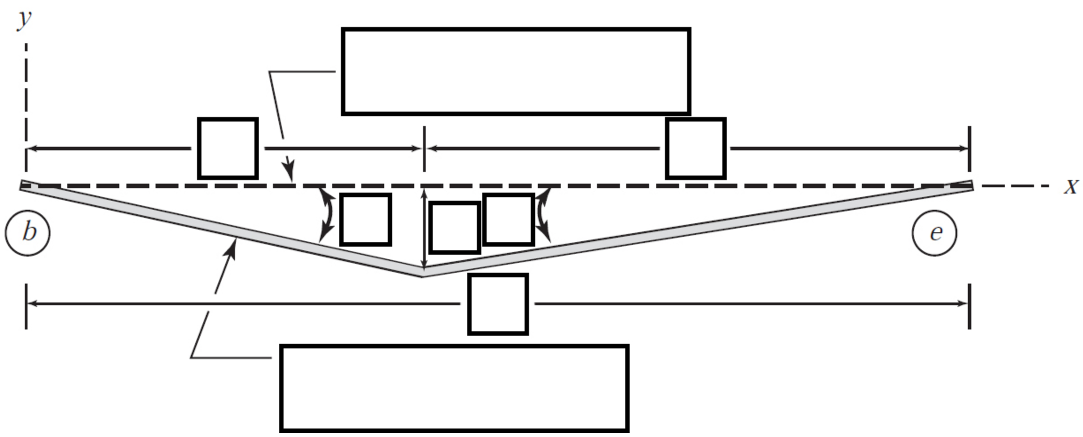
1) Fabrication error is terminology used to refer to a small ______________________________ ____________________________________ of a member in an unstressed state.
2) Similar to temperature changes, the member fixed-end forces (______) can be derived for fabrication errors.
3) Focus here is on two common types:
    a. ______________________________________________________________________.
    b. ______________________________________________________________________.
4) Figure 3 highlights the first type of fabrication error.
    a. Fabrication error results in the initial length being slightly longer than the _________ ________________________________.
    b. As the distance between the supports is _______, the compressive axial force of magnitude ____________________________________________ to reduce the length of the member from ______________________ to _________________.
    c. Therefore the fixed-end force for a member in which the fabricated length is too long can be expressed as:

    d. This equation is also valid for _______________________________ due to fabrication errors in length.

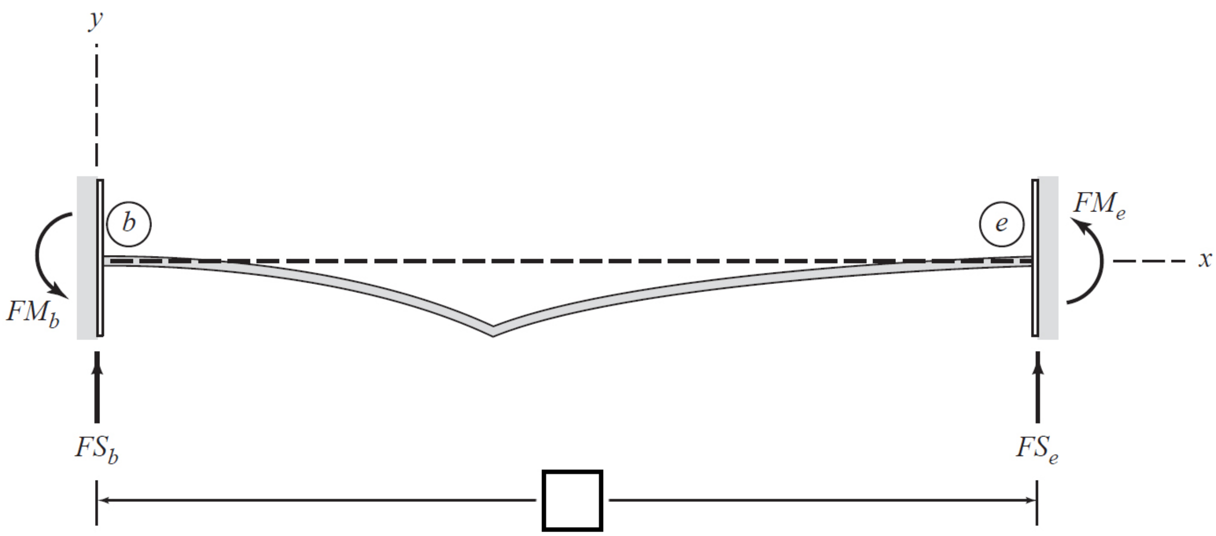

Figure 3. Plane frame member subjected to a fabrication error: (___________________________).

1) Figure 4 highlights the second type of fabrication error.
   a. Fabrication error results due to members of beams and frames having a ________ ________________________________.
      i. The example beam member is fabricated with an ______________________ which causes a small deflection of _______ at a distance of ____ from the member's left end.
   b. To determine the fixed end forces, let's express the member end rotations as:
   c. Now let's write the local end displacement vector due to the fabrication error:
   d. The rotation at ____ is negative due to the sign-convention (______________).




(a) Unstressed member with fabrication error.




(b) Fixed member with fabrication error.

Figure 4. Beam member subjected to a fabrication error.

e. Multiplying by the negative of _____, one can find the fixed-end forces for beam member as:

$$\begin{bmatrix} FS_b \\ FM_b \\ FS_e \\ FM_e \end{bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

f. Therefore the fixed-end forces for a beam can be summarized as:

g. As this fabrication error is assumed to be ____________, no _________________ _____________________ or __________________________ develop in the member.

i. Therefore this relationship holds true for __________________________.

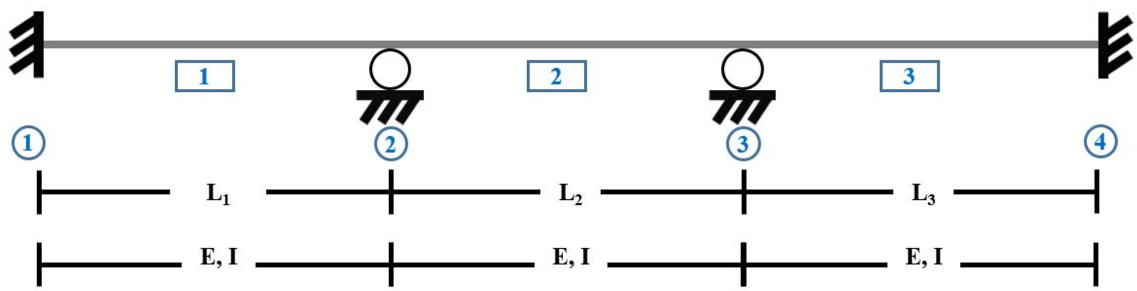
**Procedure for Analysis:**

1) Procedure for analysis is similar to that in the previous notes.
2) Temperature and fabrication errors are accounted for typically in the ________________ _______________________________________________________.
3) If a plane truss is considered, the relationships must be updated such that:

## Non-Mechanical Loading: Example

### Example #1

For the continuous beam illustrated below, determine the joint displacements, member end forces, and the support reactions. Use the matrix stiffness method. The beam is subjected to a linearly varying temperature increase on all members, where the top of the beam is increased by 55° C and the bottom of the beam is increased by 5° C. Let d equal the depth of the beam member.




**Additional Information:**

Nodes 1 and 4 are fixed

No mechanical loading

$L_1$ = 7 meters; $L_2$ = 7 meters; $L_3$ = 7 meters; d = 300 mm

I = $145(10^6)$ mm$^4$; E = 200 GPa;

$T_t$ = 55° C; $T_b$ = 5° C; $\alpha = 1.2(10^{-5})$/° C

### Solution:

Identify the number of degrees of freedom. NDOF = ________.

Identify the number of reactions. NR = ________.

E = ______________.

I = ______________.

## Finding the stiffness matrix for the members.

L = ____________; MT = ______________________; [T] = ___________________________;

$$[k]=\begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

$$[k]= \qquad\qquad = \begin{bmatrix} 1014.6 & 3551 & -1014.6 & 3551 \\ 3551 & 16571 & -3551 & 8285.7 \\ -1014.6 & -3551 & 1014.6 & -3551 \\ 3551 & 8285.7 & -3551 & 16571 \end{bmatrix} [kN, m]$$

Code Numbers:

Member 1: ____ ____ ____ ____

Member 2: ____ ____ ____ ____

Member 3: ____ ____ ____ ____

## Find the temperature change induced fixed forces.

$$T_t = \_\_\_\_\_\_\_\_\_\_; \quad T_b = \_\_\_\_\_\_\_\_\_\_$$

$$FM_{bT} = \quad FM_{eT} = \qquad\qquad = \_\_\_\_\_\_\_\_\_\_$$

## Assemble the member fixed-force vectors.

**Assemble the Structure Fixed-Joint Force Vector due to Temperature Change.**

$$\{P_f\} = \begin{Bmatrix} Q_{f_4}^{(1)} + Q_{f_2}^{(2)} \\ Q_{f_4}^{(2)} + Q_{f_2}^{(3)} \end{Bmatrix} =$$

**Find the Structure Stiffness Matrix and Joint Displacements.**

$$\{P\} = [S]\,\{d\} + \{P_f\}$$

**Compute Member End Forces.**

$$\{Q\} = [k]\,\{u\} + \{Q_f\}$$

**Compute the Reactions.**